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ELECTROMAGNETIC PROPERTIES AND THE QUANTUM HALL EFFECT OF QUASI-TWO-DIMENSIONAL ELECTRON SYSTEMS

by

VIÐAR GUÐMUNDSSON

(C)

A THESIS

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Y. Takahashi
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Date 17 May 1985

ABSTRACT

This work is concerned with the electromagnetic properties and the quantum Hall effect of quasi-two-dimensional (q-2D) electron systems. A model of the q-2D electrons is introduced by deriving the effective 2D Hamiltonian via a dimensional reduction from the 3D Hamiltonian for the electrons. The Ward-Takahashi (W-T) relations are derived for the exact density and current response functions to ensure that the initial symmetry of the model will be reflected in later approximations of these functions.

The retarded transverse current-current response functions of an ideal q-2D electron gas at zero temperature with and without an external magnetic field are calculated. Introducing a new approximation scheme which allows one to take account of the reaction of the electromagnetic field to the induced current, i.e. the influence of the electronic induced current on the dynamics of the electromagnetic field, within the framework of the conventional linear response theory in a self-consistent way, a possible form of the dispersion relation for the transverse plasmon in the system is obtained. It is found that the energy of the plasmon has a gap at $k = 0$, if the thickness of the system is finite, in contrast to the longitudinal case.

A microscopic theory of the integer quantum Hall effect (IQHE) based on the canonical equation of motion for the current operators, in the presence of impurities and the Coulomb interaction, is presented. When the appropriate thermodynamic variable is taken to be the chemical potential, due to electron reservoirs, rather than the number density, the obtained formula gives the Hall conductivity as a function of the gate voltage and shows good agreement with the MOSFET experiment by K.v. Klitzing

et. al. The formula also gives the temperature dependence quantitatively and shows explicitly how the plateaus disappear at higher temperature giving the classical Hall effect results. In connection with the fractional quantum Hall effect it is shown that the Green's function assumed by R. Tao and D.J. Thouless, in their many body model of the fractional effect, violates the fundamental requirement of current conservation.

An exact Hamiltonian is derived for the Laughlin's wavefunctions. This Hamiltonian is shown to represent non interacting electrons in contrast to the findings of S.M. Girvin et. al.

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CHAPTER 1

1.1 INTRODUCTION

Quantum field theoretical methods have been applied to many and diverse physical systems. Nowhere has as much experimental knowledge been gathered as in condensed matter physics, which can therefore serve as an ideal test ground for modern quantum field theoretical many-body (QFTMB) theories.

In particular it has been realized lately that a certain kind of condensed matter systems, the so called quasi-two-dimensional (q-2D) ones, may be of paramount interest to the development of QFTMB methods. This is due to their uniquely variable parameters in contrast to their 3D counterparts. Although the electro-magnetic properties of any condensed matter system depend on the transverse response function the dynamic transverse response function of a q-2D electron gas has not been calculated. In this thesis it will be evaluated and used to derive the dispersion of a transverse plasmon in a 2D electron gas with or without an external magnetic field.

A general microscopic model of the IQHE (integer quantum Hall Effect) will be presented which accounts for both the quantum and the classical effect. This model is independent of the detailed mechanism for electron localization and includes the spin and valley splitting and allows for the width of the landau levels.

The experimental results of K.v. Klitzing (K.v. Klitzing, 1980) will be reproduced in detail by this model, and the implications to the FQHE (fractional quantum Hall Effect) will be discussed.

In connection with exact QFTMB relations, as the Ward-Takahashi

relation and the f-sum rule, it will be shown that the only real QFTMB model of the FQHE, the Tao-Thouless model (R. Tao, 1983), does not obey the fundamental requirement for the conservation of the current.

1.2 TWO-DIMENSIONAL SYSTEMS IN NATURE

1.2.1 Experimental Systems

Quasi-2D electron systems are characterized by the fact that the electrons are almost free to move in a plane, while their motion in the direction perpendicular to the plane is severly restricted by a confining potential. So, as will be shown below, the dynamic behavior of these systems approaches that of 2D ideal systems.

If the confining potential is narrow enough the motion in the 3rd direction will be quantized into discrete energy levels; these will be resolved if the thermal and scattering broadening do not lead to overlapping of levels.

When a magnetic field is applied perpendicular to the free electron plane the free motion in the plane is quantized into discrete Landau levels (L.D. Landau, 1977) with a harmonic oscillator type spectrum.

Many electron systems have been found to exhibit a 2D behavior, and in most of them the electron density can be varied over many orders of magnitude.

Electrons can be held by an image potential to the surface of liquid Helium, (T. Ando, 1982a) (M.W. Cole, 1974) (C.C. Grimes, 1978) creating a thin layer of high mobility at low temperature. Their density can be modulated by electrodes below and above the sample, creating the so-called holding or pressing field.

2D electron system can also be created at the SiO_2-Si interface in MOSFET's by bending the conduction band of the Si with a strong positive gate bias which at the same time controls the 2D density of electrons in the so called inversion layer formed at the interface. (T. Ando, 1982a).

Another system of increasing importance in modern technology, due to its high electron mobility, is in a GaAs heterostructure. In this system the electrons are confined to the GaAs-$Al_xGa_{1-x}As$ interface by the natural conduction band discontinuity. The electron density is controlled by the donor concentration in the $Al_xGa_{1-x}As$ layer, or less commonly, by a gate bias in a FET structure (T. Ando, 1982a) (H.L. Störmer, 1984).

Many other 2D electron systems are known and the interested reader may be referred to the review article of T. Ando et. al. (T. Ando, 1982a). The three systems mentioned above are the most important ones with respect to the topics of this theses - the QHE (quantum Hall Effect) and the electromagnetic properties of 2D systems.

In the beginning of the 3rd chapter a thorough description of the structure and parameters of two of these 2D electron systems will be given, i.e. the MOSFET and the GaAs systems.

1.2.2 Two-Dimensionality

The 2D behavior of these electron systems can best be tested by comparing some dynamical characteristic length (such as the fermi wavelength or the cyclotron radius, etc.) to the width of the confining potential, or by observing some dynamical phenomenon which is distinctly different in ideally 2D or 3D systems.

The former suggestion can be carried out by comparing the characteristic spatial extention (the average distance of the electrons from the

interface (F. Stern, 1982)) of these q-2D systems (30-100A° for one level occupied of the confining potential) to the fermi wavelength.

$$\lambda_F = \frac{2\pi}{k_F} \quad , \quad k_F^2 = \frac{2\pi n_e}{g_\nu} \tag{1.1}$$

where n_e is the 2D electron density and g_ν is the inherent degeneracy. For typical densities in MOSFET's and GaAs Hetero-structures ($n_e \sim 10^{11}$ cm^{-2}) one finds that λ_F can be more than 10 times larger than the spatial extent of these systmes in the 3rd direction (T. Ando, 1982a). Therefore one strongly expects that these systems would show ideal 2D behavior.

The longitudinal plasmon dispersion for an ideal 2D electron gas was derived by F. Stern in 1967 (F. Stern, 1967) and shown to be radically different from the plasmon in 3D systems. The 3D plasmon has an energy gap while the 2D one is gapless. This ideal 2D plasmon disperson has been observed experimentally in the He system (C.C. Grimes, 1976) and in a MOSFET inversion layer (S.J. Allen, 1977) (T. Ando, 1982a).

1.2.3 Quantum Behavior

As already mentioned in section 1.2.1 the quantum behavior in the 3rd direction will be manifested in discrete energy levels, but what about the electron motion in the plane in which they are free to move?

For the MOSFET and the GaAs system the average kinetic energy of the fermions at T = 0 (the nonthermal quantum energy):

$$\frac{\pi n_e \hbar^2}{2m^*} \tag{1.2}$$

where m* is the effective electron mass, is always larger than the average

thermal energy $k_B T$ (the classical energy, k_B: Boltzmann's constant) for $T<4,2K$ and attainable electron density n_e ($10^{11} \leq n_e \leq 10^{13} cm^{-2}$ in Si MOSFET (T. Ando, 1982a, p.442), $n_e \sim 10^{11}$ in GaAs (D.C. Tsui 1981c) (H.L. Störmer, 1983a)).

Therefore these systems have to be treated quantum mechanically. On the other hand the He-system does not enter into the quantum region (due to its lower n_e) until $T<0,1K$ for the highest attainable $n_e \sim 2 \cdot 10^9 cm^{-2}$ (A.P. Volodin, 1977).

1.2.4 Observables in q-2D Systems

Observables such as the energy spectrum, the 2D electron density and the effective mass have already been mentioned in this work; how are these and other quantities, relevant to this thesis, measured?

The experimental methods used to obtain the fundamental observables are mainly of two different types; optical spectroscopy (reviewed by (J.F. Kach, 1975, 1976)) and electrical conductance measurements (M.A. Paalanen, 1983) (S. Kawaji, 1975, 1976). Fortunately these two methods both overlap and compliment each other. The diagonal elements, σ_{xx} and σ_{yy}, of the conduction tensor σ exhibit an oscillatory behavior with respect to a variation of a weak perpendicular magnetic field.

This effect, the Shubnikov-de Haas effect, has been studied theoretically by Ando (T. Ando, 1974d), see also (A. Isihara, 1979a).

The conductivity σ_{xx} dips each time an integral number of Landau levels is filled as will become clear in section 2.

The period of the Shubnikov-de Haas oscillation, with respect to the inverse of the magnetic field, depends only on the 2D electron density and some fundamental constants. This effect has become one of the most

important method to measure the 2D electron density (M.A. Paalanen, 1983) (F.F. Fang, 1977, 1978).

The theoretical studies of the Shubnikov-de Haas effect have furthermore shown that one can determine the cyclotron frequency ω_c from the temperature dependence of the oscillation amplitude (T. Ando, 1974d). The cyclotron frequency depends on the effective mass m*:

$$\omega_c = \frac{eB}{m^*c} \qquad (1.3)$$

where e is the absolute value of the electronic charge, B is the magnetic field and c is the speed of light.

The amplitude at a fixed low temperature gives the scattering time τ of the electrons once the effective mass and 2D density are known (T. Ando, 1974d).

The Shubnikov-de Haas effect therefore gives the three very important parameters n_e, m* and τ, from which the mobility of the electrons can be calculated:

$$\mu = \frac{\tau e}{m^*} \qquad (1.4)$$

Direct spectroscopic measurements can be interpreted by simple quantum mechanics (L.D. Landau, 1977) to give the energy level spectra of the confining potential (T. Ando, 1975b) and their corresponding broadening (F. Neppl, 1977, 1979) for a 2D electron system not in an external magnetic field. The scattering time τ can be obtained from the level width or the broadening (T. Ando, 1982a p.621).

$$\tau = \frac{\hbar}{2\Gamma} \qquad (1.5)$$

In an external magnetic field B the cyclotron frequency ω_c can be determined giving the effective mass m*, which combined with τ gives the mobility μ(1.3) (A. Kamgar, 1974).

The spinsplitting of the Landau levels can be used to find the effective g*-factor (R.C. Miller, 1980). It is interesting to note that the spin structure of the spectra can easily be distinguished from other structure by tilting the magnetic field from the normal of the free electron plane. The spin splitting an essentially 3D phenomenon depends on the total B-field while ω_c depends only on the perpendicular component of the B-field (F. Fang, 1968) (A. Isihara, 1979).

Tilting of the magnetic field has also been used to observe the energy levels of the confining potential from the resulting coupling to the Landau levels (Z. Schlesinger, 1983).

In FET's the 2D electron density is regulated by the gate voltage V_g, and the density n_e can be approximately estimated from the geometry of the sample.

All this information about the energy spectrum of 2D electron systems can then be used indirectly to give a picture of the confining potential and to estimate the spatial extent of the system in the 3rd direction perpendicular to the interface (F.F. Fang, 1966).

1.3 THEORY OF 2D SYSTEMS

One of the difficulties in modelling q-2D electron systems arises from the fact that even though the electrons may be confined to very thin layers they do interact with a fully 3D electro-magnetic field.

This problem has been tackled in three different ways:

In calculations of properties of the q-2D system that depend only on

the Coulomb interaction, but not on the full electromagnetic field, a model of ideally 2D electrons has been assumed (F. Stern, 1967) (A. Isihara, 1976).

On the other hand, when the full electromagnetic field has to be included, two different approaches have been used. Chiu and Quinn (K.W. Chiu, 1974), using a method common for surface properties, obtained a solution of the Maxwell equations for two half spaces and matched them together with an ideal 2D system at the boundary.

Another method used later in this theses (V. Gudmundsson, 1984) (T. Toyoda, 1984a) is to couple the 3D electromagnetic field to a q-2D electron system of a finite thickness (see Chapter 2).

Once the model has been chosen the appropriate Hamiltonian together with the Nöther currents and the ground-state can be written down. But there are some further complications. The q-2D electrons not only interact with each other, but also with a periodic background potential of the crystal lattice and with a random potential of impurities and surface states, as well in the case of MOSFET's, the glassy SiO_2 adds to the randomness of the background potential (F. Stern, 1967a).

The periodic background potential is usually taken into account by the effective mass m^* and the dielectric constant ε.

The random potential outside, giving rise to scattering τ, imposes the much more difficult question of a possible electron localization (Y. Nagaoka, 1982) (N.F. Mott, 1967) (P.W. Anderson, 1958).

The question of whether to use a finite temperature or a zero temperature QFT formalism has to be answered by comparing the possible energy spectrum with the average thermal energy.

1.4 IMPORTANCE OF 2D SYSTEMS

Q-2D electron systems are of great importance both to theoretical developments in QMBFT methods and in modern technology.

When QFTMB methods are applied to condensed matter systems to evaluate quantities such as the groundstate energy, the excitation energies, polarization, etc. the results are usually expressed as an expansion in the dimensionless parameter r_s. This parameter r_s depends only on the electron density n_e and the effective mass m* (A.L. Fetter, 1971) (A.A. Abrikosov, 1963) (E.M. Lifshitz, 1980) (D. Pines, 1977) (A. Isihara, 1971) (S. Doniach, 1974) (L.P. Kadanoff, 1962).

$$r_s = \left(\frac{1}{\pi n_e}\right)^{1/2} \cdot \frac{m^* e^2}{\hbar} \qquad (1.6)$$

In most q-2D systems the 2D electron density n_e can be varied over many orders of magnitude (T. Ando, 1982a p.442) (F.M. Peeters, 1983) in contrast with 3D electron systems where the density can hardly be varied at all. In other 3D quantum many body systems such as liquid Helium the expansion parameter, a product of the density and the hard core radius, is not variable in experiments either.

Q-2D electron systems are therefore a unique testground for QMBFT methods in condensed matter, since comparison between the theory and experiments puts more stringent requirements on the model.

Q-2D electron systems also exhibit some unique 2D behavior, like strongly variable g*-factor (Th. Englert, 1978) and effective mass m* (H.L. Störmer, 1984), and of course the recently discovered quantum Hall effects that have caught a lot of attention, (K.v. Klitzing, 1980, 1981) (H.L. Störmer, 1984).

Most of the above mentioned phenomena are either poorly understood in detail or not understood at all.

On the more technical side the QHE, in 2D systems, has offered a resistance standard that has already surpassed other known resistance standards (K.v. Klitzing, 1980, 1981). And at the same time the QHE is the only known solid-state phenomenon that can offer a high precision determination of the fine structure constant α.

Conduction properties of the 2D electron layers in MOSFET's or GaAs-FET's are also of great technical importance in micro-electronics and computer designs of the future.

1.5 REVIEW OF PREVIOUS WORK ON RESPONSE FUNCTIONS AND THE QHE IN 2D ELECTRON SYSTEMS

A lot of work has been done on q-2D electron systems. In this section the work that bears direct relation to the topics of the thesis will be reviewed, i.e. response functions, conductivity, the IQHE and the FQHE.

1.5.1 Response Functions

Response functions can be used to derive many properties of a system, such as; collective modes, ground- and excited -state energies, the effective g-factor and magnetic susceptibilities, etc. (A.A. Abrikosov, 1963) (A.L. Fetter, 1971) (A. Isihara, 1971) (E.M. Lifshitz, 1980). The main emphasis here will be put on their use to study the collective modes in q-2D electron systems.

F. Stern (F. Stern, 1967) was one of the first to calculate the density response function of an ideal 2D electron gas using the RPA (Random Phase Approximation). He then used the response function to

derive the longitudinal 2D plasmon dispersion at zero temp. and without an external magnetic field.

$$\omega = \frac{2\pi n_e e^2}{m^*} k + \frac{3}{4} \left(\frac{\hbar k_F}{m^*}\right)^2 k^2 \qquad (1.7)$$

Where ω is the frequency and k is the wave vector, k_F is the fermi wave-vector.

The main difference between the 2D plasmon and the 3D one is that the 2D dispersion is gapless, i.e. $\omega \to 0$ as $k \to 0$. This can be used, as mentioned before, to determine whether a system is dynamically 2D or not. This characteristic 2D behavior of the plasmon has been confirmed, to the lowest order, in experiments with electrons bound to the surface of liquid He (C.C. Grimes, 1976) and in experiments on MOSFET inversion layers (S.J. Allen, 1977).

A. Isihara and T. Toyoda (A. Isihara, 1976) evaluated the density response function at finite temperature for an ideal 2D electron gas and used it to find the exchange energy.

A.K. Rajagopal (A.K. Rajagopal, 1977) included exchange diagrams in addition to the RPA diagrams in his derivation of the 2D plasmon dispersion at T=0. He claims that the exchange terms result in a correction to the quantum term of the dispersion (1.5) (the second term on the r.h.s.).

Rajagopal also derived the real part of the transverse current response function in the static ($\omega \to 0$) and long wave length ($k \to 0$) limit and used it to evaluate the orbital magnetic susceptability.

G.F. Giuliani and J.J. Quinn (G.F. Giuliani, 1984) calculated the 2D plasmon damping at T=0 due to impurity scattering. Their calculation

is performed in the high and low impurity concentration limit assuming a δ-impurity.

All these calculations of the plasmon dispersion mentioned above have been done assuming only one subband of the confining potential to be full. There have been some attempts to evaluate the plasmon dispersion for systems with more than one subband occupied.

Y. Takada (Y. Takada, 1977) calculated the density response at finite temperature, including more than one subband. In addition to the "usual 2D plasmon" he finds another collective mode, the acoustic plasmon (when the electrons of each subband oscillate out of phase).

Intra and inter subband plasmons have been studied by S. Das Sarma (S.D. Sarma, 1984) who maintains that the interference of these modes explains the observed plasmon mass increase at higher wave numbers in MOSFET's (D. Heitman, 1982).

The longitudinal plasmon has also been studied in super lattices, i.e. many 2D electron systems stacked together (G. Qin, 1983) (A.C. Tselis, 1984).

The collective oscillations also exist in a 2D electron gas in an external perpendicular magnetic field.

The first calculation of the current response function for a 2D electron gas at finite temperature, and in a strong magnetic field, was performed by K.W. Chiu and J.J. Quinn (K.W. Chiu, 1974). Using the RPA and a high temperature semiclassical approximation they were able to calculate numerically the dispersion of the magneto-plasmon for long and short wavelengths.

This calculation has been improved by M.L. Glasser (M.L. Glasser, 1983) who derived the density response at finite temperature in a closed

integral form over a finite range. He claims to have obtained terms neglected in the semiclassical approximation of J.J. Quinn.

C. Kallin and B.I. Halperin use a combination of the RPA and the ladder approximation to calculate the density response of a 2D electron gas with n full landau levels at T=0.

They show that the collective mode can be thought of either as a magneto plasmon or as a magnetic excitation (i.e. a bound state of a hole in a filled Landau level and one electron in an otherwise empty Landau level). They obtain, numerically, dispersion curves and also the spin-wave dispersion for the lowest landau level. The implications to the FQHE are discussed.

A.H. MacDonald (A.H. MacDonald, 1984b) uses the Hartree-Fock approximation for a 2D electron gas, with an integral number of fully occupied Landau levels, to evaluate the spin dependent response function at T=0. He uses numerical methods to evaluate the dispersion curves and finds that the collective modes are magnetoplasmons or excitations between Landau levels, in agreement with the results of B.I. Halperin.

None of the above mentioned works have dealt with the possible transverse plasmon in 2D electron systems, though J.J. Quinn correctly states (K.W. Chiu, 1974) that the longitudinal magneto plasmon has to be derived with transverse effects in mind. In only one paper are claims laid to have derived the dispersion of transverse excitations in a 2D electron gas (G.K. Agarwal, 1981).

G.K. Agarwal et. al. use a numerical simulation of a classical 2D electron gas to obtain the dispersion of longitudinal and transverse excitations. The physical meaning of this transverse mode is far from clear.

As mentioned before, the response functions have also been used to calculate other interesting quantities of 2D systems such as the density dependence of g* and m* (A. Isihara, 1979, 1980, 1983).

1.5.2 The Importance of Transverse Response Functions and Plasmons

The electromagnetic properties of any condensed matter system are related to the transverse current response function i.e. the propagation of an electromagnetic wave or a transverse plasmon mode, etc. depend on the transverse current response and the Maxwell equations.

The transverse plasmon was first derived in 3D electron systems by D. Bohm and D. Pines (D. Bohm, 1951). They used a considerably complicated canonical transformation in order to calculate the dispersion.

H. Matsumoto et. al. (H. Matsumoto, 1980) applied their quantum electrodynamics of solids to the same problem and obtained the dispersion relation of the 3D transverse plasmon in agreement with Bohm and Pines.

Nobody has calculated the dynamic transverse response function of a q-2D electron gas with or without an external magnetic field. In this theses the current response function will be calculated for the 2D electron gas. The longitudinal and the transverse part will be separated in a general way. The transverse response function will be coupled to the 3D classical electromagnetic field according to the new self consistent linear response method, leading to the transverse plasmon dispersion relation.

It will be shown that our simple method, in the case of 3D electrons leads to the same results (including the first order quantum correction) as H. Matsumoto et. al. have obtained using the second quantized form of the electromagnetic field.

The transverse response of 2D electron systems may be of importance for the FQHE, for when the magnetic field increases in strength the current becomes almost entirely transverse.

The transverse plasmon dispersion may be of significance in the proposed micro-optical systems of the future. Recently the superconductivity of q-2D electron systems has attracted great attention (H. Takagi, 1982a, b, 1983). In these systems the transverse current response connects the behavior of the current and the electromagnetic field in the Meissner effect. Our formalism might therefore be helpful to shed some light on the fascinating subject of q-2D superconductivity.

1.5.3 Conduction

The conductivity tensor σ for a 2D electron gas is derived from the current response function of the system, calculated perturbationally with respect to the impurity Hamiltonian (S. Doniach, 1974).

The irreversibility enters the problem when a random average is taken over the impurity sites and a selective group of Feynman-graphs is used for the perturbation calculation (A.A. Abrikosov, 1963) (A. Isihara, 1971) (S. Doniach, 1974).

The emphasis of conductivity calculations shifted with the discovery of the QHE, so in this section the pre-QHE studies will be reviewed.

Calculations of the conductivity in 3D condensed matter systems were pioneered amongst others, by R. Kubo (R. Kubo, 1957), using linear current response with respect to an external electric field E in the presence of impurities. R. Kubo later on (R. Kubo, 1969) founded the so called "center migration theory", in which the cyclotron motion is separated from the diffusive motion of the electrons, in order to calculate the conductivity

of a 2D electron gas in an external magnetic field B.

Q.F.T. methods, later used in 2D systems to calculate the conductivity, where laid down in 3D systems by H. Shiba et. al. (H. Shiba, 1970, 1971) and H. Hasegawa (H. Hasegawa, 1969).

T. Ando used these QFT methods for 2D electrons (T. Ando, 1974a,b,c) in order to derive the Landau level broadening (the imaginary part of the self energy) consistently using either δ-impurities or longer range impurities represented by a Gaussian potential well. These results were then used to evaluate σ_{xx} at finite temperature in various approximations - the self consistent Born approximation (SCBA), the single site approximation (SSA) or the many site approximation (MSA). All but the most simple results have to be presented by numerical means.

These calculations were carried out in the strong magnetic field limit, so only one Landau level is accounted for.

T. Ando (T. Ando, 1974d) also considered the more difficult problem of a 2D systems in an arbitrary magnetic field, and thereby derived an expression for the very important Subnikov-de Haas effect (see sec. 1,2,4).

In 1975 (T. Ando, 1975) calculated the off-diagonal conductivity σ_{xy}:

$$\sigma_{xy} = -\frac{n_e ec}{B} + \Delta\sigma_{xy} \qquad (1.8)$$

obtaining the well known classical result in addition to the higher order correction $\Delta\sigma_{xy}$. The higher order corrections to both σ_{xy} and σ_{xx} are usually not small and suffer from problems with analyticity of the self energy.

Ando never discussed in his paper what quantities would be measured in a Hall effect experiment (see Section 2.3) consequently he was not

able to see the QHE.

The conduction calculations of T. Ando led the group of S. Kawaji and J. Wakabayashi et. al. (J. Wakabayashi, 1976, 1978, 1980a) (S. Kawaji, 1975, 1976) to do measurements on MOSFETS at low temperature (T~1.4K) and high magnetic field (B<150 KG).

They obtained a good qualitative agreement for σ_{xx} and σ_{xy} with Ando's theory, but they did not find the plateaus that later were known to characterize the IQHE. This may be due either to their sample having too low mobility, or to their preoccupation in studying σ_{xx} and σ_{xy} in connection with Ando's theory, instead of using ρ_{xx} and ρ_{xy} directly (ρ: resistivity tensor). In this connection it is interesting to see that Th. Englert and K.v. Klitzing (Th. Englert, 1978) did a similar experiment on a MOSFET at T = 1.5K and B = 142 KG. They find both ρ_{xx} ρ_{xy} and σ_{xx} σ_{xy}, and their ρ_{xy} shows the beginning of the formation of the flat regions, which later were identified as the signature of the IQHE. They also find evidence for oscillations of the effective g-factor.

Much attention has been devoted to the difficult question of Anderson localization of 2D electrons in impure systems, and to its connection to the conductivity.

Several papers have been written on whether all electrons in 2D systems are localized or not (E. Abrahams, 1979) (L.P. Gor'kov, 1979) (B.L. Altshuler, 1980). This controversial topic will be further addressed in section 1.5.3 on the QHE together with further developments in the conductivity calculation after the discovery of the QHE.

1.5.4 IQHE

The IQHE has been hailed as one of the most important phenomenon

discovered in quantum condensed matter systems.

It had not been predicted by any theory, in spite of the active research on 2D electron systems.

Good review articles on the QHE have been published (K.v. Klitzing, 1981) (S. Kawaji, 1983) (H.L. Störmer, 1984). For description of the experimental systems and the results see Chapter 3.

The IQHE is manifested by flat plateaus in ρ_{xy} and near vanishing of ρ_{xx} when an integer number of Landau levels is filled in a 2D electron system in a strong magnetic field and at a low temperature (T<4K). The value of $\sigma_H (= 1/\rho_{xy})$ is quantized to an integer times the constant e^2/h, where h is Plancks constant and e is the absolute value of the electronic charge. (h/e^2 is therefore a natural resistance unit). The QHE was first discovered by K.v. Klitzing et. al. (K.v. Klitzing, 1980) who found that the plateaus of ρ_{xy} are flat to one part in 10^5 and ρ_{xx} vanishes at the same time. This experiment was conducted on a MOSFET at T = 1.5K and a constant magnetic field B. The gate voltage V_g was varied in order to change the density of electrons and populate the successive Landau levels.

The spin and valley splitting is clear for the lowest two Landau levels.

K.v. Klitzing suggested that the IQHE could be used to determine the fine structure constant α:

$$\alpha = \frac{e^2}{\hbar c} \qquad (1.9)$$

from the high precision value for ρ_{xy}:

$$\rho_{xy} = \frac{h}{e^2 i}, \quad i = 1,2,..... \qquad (1.10)$$

(K.v. Klitzing, 1980, 1981).

Soon after, D.C. Tsui et. al. (D.C. Tsui, 1981 a,c) discovered the IQHE in GaAs heterostructures and found ρ_{xy} to have plateaus flat to one part in 10^6 at T = 4.2K. The universality of the QHE had been shown by these two different sample materials and by samples of different shapes and sizes (K.v. Klitzing, 1981).

H.L. Störmer (H.L. Störmer, 1983b) further boosted the universality of the QHE by observing it in a 2D hole system in a GaAs heterostructure, thereby showing that the effect is independent of the details of the band structure.

K.v. Klitzing et. al. (K.v. Klitzing, 1982) studied the IQHE at temperatures down to 8mK while M.A. Paalanen et. al. (M.A. Paalanen, 1982) reached a temperature of 50 mK. Both these groups find the steps in ρ_{xy} to be extremely sharp and to have reached almost full possible plateau width. The spin splitting is observed for the first two Landau levels.

The highest precision in determining the flatness of the plateaus in ρ_{xy} has been obtained by J. Kinoshita et. al. (J. Kinoshita, 1983) who claims that they are flat to one part in 10^7 in a MOSFET.

The temperature dependence of the IQHE has been studied by several groups. In particular K.v. Klitzing (K.v. Klitzing, 1981) shows how the plateau width increases with decreasing temperature. While M.E. Cage et. al. (M.E. Cage, 1984) have observed a shift of the value of ρ_{xy} away from the precisely quantized one (1.8) with increased temperature. The

latter emphasized the difference between this shift and the temperature dependent slope of the plateaus, which they quote from an unpublished measurement of K.v. Klitzing et. al.

The plateau width of ρ_{xy} is also dependent on the mobility of the 2D electrons in the sample. J.E. Furneaux and T.L. Reinecke (J.E. Furneaux, 1984a, b) and D.A. Syphers et. al. (D.A. Syphers, 1984) have studied the effect of the impurity concentration on the plateau width, by drifting Na^+ ions into MOSFETS. They find that the plateau width increases with added impurities. As an explanation they suggested that the region of localized states in the tails of the Landau levels grow, with increasing impurity concentration, and thereby narrow the region of extented states at the Landau level center (see the next section on IQHE theories). They also observe that the spin plateaus grow slower in width, which may possibly be due to overlapping of the spin split Landau levels.

IQHE Theory

Many theories have been put forward to explain the IQHE and can be loosly grouped into four categories: localized-states-theories, gauge-principle-theories, Reservoir-theories and theories based on a chiral anomaly.

H. Aoki and T. Ando (H. Aoki, 1981) built a model of the IQHE using the previous conduction calculations of T. Ando (T. Ando, 1975) in connection with the idea of localized states. They claim that the impurities in the 2D electron system cause localized states of electrons to be in the tails of the Landau levels and between them.

Furthermore when σ_{xx} and $\Delta\sigma_{xy}$ are evaluated the main contribution

comes from the states around the Fermi level. Therefore if the Fermi level is in the localized states then σ_{xx} and $\Delta\sigma_{xy}$ vanish, leading to:

$$\sigma_{xy} = -\frac{n_e ec}{B} \qquad (1.11)$$

It is then argued that the Coulomb interaction has no effect due to the particle hole symmetry.

They have a harder time explaining how the density n_e in (1.9) is the total density of electrons in the system, not only the density of mobile electrons; this is supposed to be due to the higher speed of the extended electron states to compensate for the carrier loss to the localized states. This explanation is far from being convincing, and it should be kept in mind that the essential condition for the Hall effect in general to be observed, that either the perpendicular current I_y or the perpendicular voltage V_y has to vanish, has not been used by H. Aoki and T. Ando.

R.E. Prange (R.E. Prange, 1981) uses a single δ-impurity (causing localized states) in order to evaluate this current compensation.

Much work has been done on Anderson localization, random impurity potential and periodic potential (D.J. Thouless, 1981) (D.C. Tsui, 1981) (T. Ando, 1982a) (N. Hoshi, 1982) (P. Středa, 1982b) (G.F. Giuliani, 1983) (D. Yoshioka, 1983a) (H. Levine, 1983, 1984a, b, c) (D. Yoshioka, 1983a) (J.E. Avron, 1983) (J.T. Chalker, 1984) (D.E. Khmelnitskii, 1984) (S. Hikami, 1984) in connection with IQHE model, but this whole field is still quite controversial.

R. Joynt (R. Joynt, 1984) finds in his research that all physically reasonable disordered potentials should exhibit the IQHE in the limit

of a strong B-field. But he also claims that the mechanism of localization is qualitatively different from that of Anderson localization. His article is a good review of earlier work on the localization problem.

A very similar approach, percolation theory, has also been used to model the IQHE (R.F. Kazarinov, 1982) (S. Luryi, 1983) (S.A. Trugman, 1983). The main results of the percolation theory has been that there is always a narrow band of extented states at the center of each Landau level.

A very different approach to the IQHE, but still used in close connection with localization, have been the so called fundamental gauge principle theories that were first introduced by R.B. Laughlin (R.B. Laughlin, 1981). In this theory the electrons are located on a cylindrical ribbon, with magnetic field both through its center and sides (a Bohm-Aharonov geometry). Due to the extended states a gauge transformation would have to be periodic (in the circumference) in order to leave the system gauge invariant. The total energy of one electron depends on its location with respect to the electric field across the ribbon. The gauge transformation changes this location, so in order for the total energy of the system to remain gauge invariant an integral number of electrons has to be moved from one edge of the ribbon to the other. This leads to the quantization of ρ_{xy} due to the relation between the current along the ribbon to the voltage across it caused by the magnetic field.

The weakness of this theory is that it is basically a one electron theory, so no attention is given to the antisymmetrization of fermionic many-body systems. This theory is therefore equally good for boson systems.

Another problem is the special geometry of the sample required. In most experiments of the QHE a thin elongated flat sample is used and it is not obvious how this theory can be extended to such samples.

R.B. Laughlin did not specially account for impurities in the system, but assumed that always some extended states would be present. This point has been elaborated by B.I. Halperin (B.I. Halperin, 1982) who uses a physical argument in order to prove that there should always be some extended states present. Furthermore, he indicated that extended edge states are of importance if the Fermi energy E_f is not the same on both sides of the ribbon.

For additional work along these lines see (H. Aoki, 1982) (R. Rammal, 1983) (A.H. MacDonald, 1983f). The redistribution of charges which generate the Hall voltage are discussed by A.H. MacDonald (A.H. MacDonald, 1983b).

The importance of an electron reservoir has been stressed in a paper by G.A. Baraff and D.C. Tsui (G.A. Baraff, 1981). They emphasized that for σ_{xy} (1, 11) to have plateaus, as a function of B, n_e must vary with B, so there must be a reservoir of electrons.

In GaAs heterostructures the Si donors in the $AL_xGa_{1-x}As$ layer, separated by a thin layer of undoped $AL_xGa_{1-x}As$ from the 2D electrons, can act as a reservoir.

A model built on the assumption that the Si donors are the only reservoir in GaAs is reported, by G.A. Baraff and D.C. Tsui, to be able to account for 78% of the plateau width in an experiment by D.C. Tsui et. al. (D.C. Tsui, 1981a, b). The rest of the plateau width may have been caused by localized states or interface states.

This theory is very appealing, having in mind that it may explain the much narrower plateaus in Si-MOSFETs due to the fact that no

reservoirs are known in MOSFETs comparable to the ones in GaAs heterostructures.

The role of the localized states may also have to be reconsidered in light of experiments (J.E. Furneaux, 1984a, b) (D.A. Syphers, 1984) that show the plateau width increased with added impurities.

M.A. Paalanen (M.A. Paalanen, 1983) states that this model can only account for 30% of the plateau width in his precision experiment.

There is no doubt though that reservoirs are of fundamental importance, but the mechanism has to be studied more thoroughly.

The plateau width has also been studied by A. Isihara (A. Isihara, 1983) who found it to depend on the ratio of the field energy and the thermal energy.

O. Heinonen and P.L. Taylor (O. Heinonen, 1983) constructed an essentially one electron model showing that the plateaus can exist with no impurities present.

Several authors dealt especially with various aspects of the conduction.

P. Středa (P. Středa, 1982a) derives a quantum correction to the classical σ_{xy}, differing substantially from the "center migration" results of R. Kubo (R. Kubo, 1969).

A.H. MacDonald (A.H. MacDonald, 1983a) claims to show that the simple classical result for the quantum Hall conductance can also be derived for a relativistic system using the Dirac equation.

And the vanishing of all higher order corrections to the Hall conductance, due to the external electric field, is proved by Q. Niu and D.J. Thouless (Q. Niu, 1984a).

O. Heinonen and P.L. Taylor (O. Heinonen, 1984) concluded that the

breakdown of the IQHE, observed when the current along the sample I_x is increased (M.E. Cage, 1983), is due to electron phonon interactions.

Recently a quite different theory of the IQHE has been presented by K. Ishikawa (K. Ishikawa, 1984). He claims that a possible chiral anomaly of electrons described by the Dirac equation may be responsible for the IQHE; he also points out that this theory may act as a microscopic theory for the gauge theory of Laughlin (R.B. Laughlin, 1981).

1.5.5 General Microscopic IQHE Model

As stated earlier T. Ando et. al. calculated the conductivity tensor σ (T. Ando, 1975) for q-2D electrons in an external magnetic field.

H. Aoki and T. Ando (H. Aoki, 1981) then try to show that σ_{xy} develops plateaus due to higher order corrections $\Delta\sigma_{xy}$ when an integer number of Landau levels is filled.

The problem with this approach is that they never take into account the important pre-condition for the Hall effect, that either the Hall current I_y or the Hall voltage V_y has to vanish.

Unfortunately many experimental groups have also tried to apply the Aoki-Ando theory directly to their experiments, and discussed the effects of $\Delta\sigma_{xy}$ (S. Kawaji, 1975, 1976) (J. Wakabayashi, 1980) (K.v. Klitzing, 1981).

In this theses it will be emphasized that there are two types of experiments. In the more common one the condition $I_y = 0$ holds, and it will be shown that ρ_{xy} should exactly equal:

$$\rho_{xy} = \frac{B}{n_e ec} = \sigma_H^{-1} \qquad (1.12)$$

While in experiments where $V_y = 0$ σ_{xy} exactly equals:

$$\sigma_{xy} = -\frac{n_e ec}{B} + \Delta\sigma_{xy} \tag{1.13}$$

with a correction term $\Delta\sigma_{xy}$ depending on σ_{xx}. It should be stressed that ρ_{xy} in (1.12) is independent of the value assumed by ρ_{xx} at the same time.

It will be shown that the conductivity tensor can be derived from an equation of motion for the 2D interacting electron current, quantum mechanically, analogous to the classical derivation.

The Hall conductance will have the simple classical form:

$$\sigma_H = \frac{I_x}{V_y} = -\frac{\langle n_e \rangle ec}{B} \tag{1.14}$$

where $\langle n_e \rangle$ is the grand canonical average of the 2D electron density n_e. In the evaluation of $\langle n_e \rangle$ it will be essential to consider the anti-symmetry property of the many electron wave functions in contrast to R.B. Laughlin's (R.B. Laughlin, 1983) essentially one electron gauge principle theory where the fermionic property of the electrons is not accounted for.

The general model of the IQHE, proposed by us, is able to handle different plateau widths, temperature, spin and valley degeneracy, and is general enough so as to be independent of the exact mechanism for localization and electron reservoirs.

In the third chapter the excellent reproduction of experimental data by this model will be demonstrated and implications for the FQHE and further work on the IQHE will be discussed.

1.5.6 FQHE

The FQHE was surely the most unexpected and fascinating phenomenon discovered in condensed matter physics in recent years.

It has also not yet been understood, since to date no adequate microscopic theory of the FQHE has been presented.

After the discovery of the IQHE the group of D.C. Tsui (D.C. Tsui, 1982) was looking for the elusive Wigner crystalization of a 2D electron gas in a GaAs heterostructure, expected at high enough magnetic field so only one Landau level would be partially occupied.

What they found instead was a dip in ρ_{xx} and a plateau in σ_H ($= 1/\rho_{xy}$) at one third of the value e^2/h when the temperature was below 1K. The plateau was measured to be flat to one part in 10^2 at T = 0.42K.

Soon after (H.L. Störmer, 1983), the fractional effect was seen both at filling factors ν = 1/3 and 2/3 with clear plateaus in ρ_{xy} and dips in ρ_{xx}. (ν: is the factor multiplying e^2/h in the expression for σ_H, i.e. $\sigma_H = \nu \frac{e^2}{h}$).

Moreover, plateaus were also observed forming in ρ_{xy}, and corresponding dips in ρ_{xx}, at ν = 4/3, 5/3, 2/5, 3/5, 4/5, 2/7. While nothing has been seen at even denominal fractional filling factors.

This experiment was carried out on a GaAs sample of high mobility, and it was noted that no fractional effect could be seen in samples with low mobility, which instead developed very wide IQHE plateaus.

The stability of the electron system, the plateaus in ρ_{xy} and the vanishing of ρ_{xx} in the FQHE indicates that there might be gaps in the energy spectra of the system at these fractional filling factors. The reappearance of finite ρ_{xx} or σ_{xx} when the temperature is increased

implies then that the conduction is thermally activated i.e. the conductivity σ_{xx} has a factor of the form:

$$\sigma_{xx} \sim \cdots \exp(-\frac{\Delta}{k_B T}) \qquad (1.15)$$

where Δ is the energy gap (the activation energy) depending on the filling factor ν, and k_B is the Boltzmann constant.

This dependence of σ_{xx} or ρ_{xx} on T has been observed in experiments. A.M. Chang et. al. (A.M. Chang, 1983, 1984) have studied systematically the low T (65-770 mk) behavior of the $\nu = 2/3$ effect in order to determine the activation energy Δ. They found the same gap from both ρ_{xx} and ρ_{xy} having the maximum value at $\nu = 2/3$ and decreasing rapidly to both sides as a function of ν. The width of the narrow activation energy peak is $\Delta\nu = 0.1 \pm 0.01$, while the maximum value at B = 92.5 KG is 0.830 ± 0.03K.

The activation energy has been measured for $\nu = 1/3$ and 2/3, also in GaAs, by S. Kawaji (S. Kawaji, 1984). The ratio of these two values does not agree with the theory of R.B. Laughlin (R.B. Laughlin, 1983b) (see below).

The universality of the FQHE has been clearly demonstrated by V.M. Pudalov (V.M. Pudalov, 1984) and D.A. Syphers (D.A. Syphers, 1983) who have studied the effect in MOSFET's, and by E.E. Mendez (E.E. Mendez, 1984) observing the FQHE in systems of 2D holes.

The question of the range of ν, for which the FQHE can be seen, has been looked into by two groups:

E.E. Mendez (E.E. Mendez, 1983) reports that no effect is seen below $\nu = 1/5$ at T = 0.068K even though the filling factor is brought down to

$\nu = 1/11$.

K.v. Klitzing has found structures developing at $\nu = 3/4, 1/2$ (K.v. Klitzing, 1984) and thereby suggests that our idea of the FQHE may have to be changed drastically in the future.

E.E. Mendez et. al. (E.E. Mendez, 1984a) finds structures at $\nu = 4/3, 5/3, 7/3, 8/3$.

From these last two accounts it is clear that the FQHE is not restricted to the lowest Landau level.

FQHE Theory

Despite the great attention focused on the FQHE no adequate theory explaining it microscopically has been presented.

Several theories were suggested soon after the discovery of the FQHE, but the surviving ones can be loosely grouped into 3 groups: Trial wave function theories, super-lattice ground state theories and fractional fermion number theories.

The experimental groups finding the FQHE (D.C. Tsui, 1982) (K.v. Klitzing, 1982) suggested that the effect could be caused by the formation of a Wigner solid or a charge-density-wavelike ground state (H. Fukuyama, 1982) (D. Yoshioka, 1983b). The main reason for the abandonment of this approach was the fact that no conduction threshold was found for low voltages, due to the pinning of the ground state to the impurities. Another reason is that some investigators (D. Yoshioka, 1983c) (R.B. Laughlin, 1983a) (W.P. Su, 1984) found that the ground state energy of few electrons (3-6) in a strong magnetic field is less than that of a Wigner solid. They also found some signs of extra stability at

$\nu = 1/3$, through it is not clear if the concept of a filling factor is appropriate for so few electrons.

R.B. Laughlin (R.B. Laughlin, 1983b) constructed a trial wavefunction for a ground state with a filling factor $\nu = 1/p$, where p is an odd integer. Through a connection with the 2D OCP (one component plasma) he found that the excitations are of a fractional charge in an incompressible quantum fluid. The ground state energy is lower than in a Wigner crystal.

F.D.M. Haldane (F.D.M. Haldane, 1983) improved R.B. Laughlin's approach by constructing a translationally invariant trial wavefunction that can also account for the FQHE at filling factors of the type $\nu = q/p$, where p is an odd integer while q can be any integer less than p.

B.I. Halperin discusses possible extensions of Laughlin's wavefunctions to $\nu = q/p$ in a review article on both the IQHE and the FQHE (B.I. Halperin, 1983). Similar extension of wavefunctions are also given by P.W. Anderson (P.W. Anderson, 1983).

The Laughlin wavefunctions are not an exact solution to the interacting electron problem (Coulomb interaction), but S.M. Girvin (S.M. Girvin, 1983) claims that they are an "exact solution" to the problem of harmonically interacting particles. (see a comment on the validity of this claim in the next section).

The Laughlin wavefunctions have been used to construct the approximate 2 point classical correlation functions in order to facilitate the comparison of the ground state energy for different models (S.M. Girvin, 1984b). The same author has also given a more rigorous derivation of the hole-excitation of the Laughlin ground state and extended it to $\nu = q/p$ (S.M. Girvin, 1984a).

P.K. Lam (P.K. Lam, 1984) and S.M. Girvin then use the correlation function to show that for $\nu \leq 1/7$ the ground state will transform from the imcompressible fluid state (described the Laughlin wavefunctions) into a Wigner solid. This theory should be compared with the experiments of E.E. Mendez (E.E. Mendez, 1983) in which no FQHE is observed below $\nu = 1/5$). At the same time it is necessary to keep in mind that the ratio of the energy gaps for $\nu = 1/3$ and $2/3$, according to the theory of Laughlin, is in contradiction with the experimental results of S. Kawaji et. al. (S. Kawaji, 1984).

There has been some disagreement over the kind of statistics that the quasiparticles of Laughlin obey. R.B. Laughlin (R.B. Laughlin, 1984) argues that the excitations must be usual fermions, while B.I. Halperin (B.I. Halperin, 1984) maintains that they must obey fractional statistics, and in that connection argues that their energy spectrum has downward cusps (stable) at odd denominal filling factors ν, and peaks (unstable) at even denominal ν.

There have not been many attempts to construct QMBFT microscopic models of the FQHE though D.J. Thouless (D.J. Thouless, 1984) expresses the commonly held view that the Coulomb interaction may be of importance.

R. Tao and D.J. Thouless have presented a QMBFT model of the FQHE in which they assume that the groundstate is a super lattice in Landau-orbital space (R. Tao, 1983a). The lifting of the degeneracy caused by the Coulomb interaction, is calculated self consistently using the RPA.

The energy gap Δ thus found between holes and particles depends on the filling factor ν and is largest for values of ν close to $1/2$. In this model the odd denominal ν are not favored over the even ones in contradiction to experimental results.

This model has some serious flaws as will be discussed later on. Tao and Thouless truncated the Landau level space such that only the lowest Landau level is included in the calculations. Y.E. Lozovik (Y.E. Lozovik, 1984) has calculated the Hall conductance $\sigma_H(\omega)$ (the ratio of the longitudinal current I_x and the Hall voltage V_y) and in his paper he discusses the importance of considering the transitions between all the Landau levels, even in strong B field, in order to preserve exact relations such as the f-sum rule.

He also stresses the importance of including the transitions between localized states and extended states in models that consider the electron localization.

In a continuation of the Tao-Thouless theory S.M. Girvin (S.M. Girvin, 1984c) formulated a general theory of quantum mechanics within the lowest Landau level. An algorithm for projecting any quantum operator on the lowest Landau level is introduced in conjunction with coherent states and path integrals.

Very recently there has been a surge of activities around a completely different model of the FQHE using the Dirac equation to describe the electrons. In these models (R. Jackiw, 1984a, b, c) the theory of fermion fractionalization due to a topologically generated groundstate is used. There seems to be difficulty to specify which fractions are stable.

There are some theories of the FQHE that do not lend themself to an easy interpretation.

R.S. Markiewicz et. al. (R.S. Markiewicz, 1983) claim that the FQHE can be explained by properly accounting for the magnetic compressibility of the Wigner lattice.

I.E. Dzyaloshinskii (I.E. Dzyaloshinskii, 1984) maintains that the FQHE can be understood qualitatively from the viewpoint of the symmetry of the electron states in an external B-field.

Q. Niu and D.J. Thouless (Q. Niu, 1984b) claim to have expressed the Hall conductivity in a topological invariant form in the presence of impurities and interactions. They also suggest that the ground state has a broken symmetry.

R. Tao (R. Tao, 1984b) gives vague arguments for the case that the FQHE may be modelled in an analogous way to R.B. Laughlin's gauge principle for the IQHE (R.B. Laughlin, 1981).

1.5.7 The Application of Exact QMBFT Methods to the FQHE

The FQHE is a great challenge to many investigators and until now no real progress has been made in explaining it. The two best known models (R.B. Laughlin, 1983) (R. Tao, 1983) suffer from some serious flaws.

It will be shown that the Laughlin wavefunctions can be written as a power of a Slater determinant of the single electron wavefunctions in the lowest Landau level, and therefore the fractional character of these wavefunctions is a trivial artifact. In this connection it will also be seen that these wavefunctions are not an exact solution of the problem of harmonically interacting electrons, in contrast to the claims of S.M. Girvin et. al. (S.M. Girvin, 1983).

If the FQHE is a manybody phenomena then one needs QMBFT methods, rather than a guess of trial wavefunctions, in order to construct a microscopic model. The only microscopic model, the Tao-Thouless model

(R. Tao, 1983) suffers from some serious inconsistencies as will be shown in Section 2.4.1. It cannot be stressed enough, for example, when condensed matter systems are modelled, how important it is to carefully consider the choice of the QFT in-fields, and the approximations used have to satisfy exact relations as the Ward-Takahashi relation and the f-sum rules.

The Greens function in the Tao-Thouless model will be shown to violate current conservation and the only possible remedy for the divergent plasmon dispersion is seen to come from the application of the f-sum rule to redefine the gap parameter Δ, in order to account for the effects of higher Landau levels.

It is an open question whether the assumed groundstate in the Tao-Thouless theory can be derived using the Coulomb interaction, and what the role of symmetry breaking is in the model.

CHAPTER 2

ELECTROMAGNETIC PROPERTIES AND THE QHE OF QUASI-2D-ELECTRON SYSTEMS

The fact that q-2D electron systems are a special limiting case of 2D systems has to be reflected by the models of the 2D electron gas. In this chapter the effective 2D Hamiltonian describing the 2D electron system will be derived, through dimensional reduction, from the 3D Hamiltonian. It will furthermore be shown that this procedure is necessary for the derivation of some of the electromagnetic properties of q-2D electron systems (such as the transverse plasmon).

In section 2.2 QFTMB methods will be used in order to study the possible propagation of electromagnetic waves in a q-2D electron gas. This study will be carried out on systems with and without an external magnetic field. Section 2.3 is devoted to the study of IQHE. A simple formula for the Hall conductivity σ_H will be derived for an interacting electron gas in the presence of impurities. This derivation is shown to parallel completely the classical derivation, even though a quantum system is considered. The resulting expression for the Hall conductivity will be seen to cover both the classical and the quantum regions of the IQHE. The theory presented here of the IQHE will be shown to be fundamentally different from the theory of R.B. Laughlin (R.B. Laughlin, 1981) and T. Ando and H. Aoki (H. Aoki, 1981).

With the better understanding of the IQHE, the attention can be turned to the FQHE. In section 2.4 it will be shown that the QFTMB model of Tao and Thouless (R. Tao, 1983) for the FQHE does violate the fundamental law of the current conservation, some remedies will be discussed.

This section concludes with the derivation of the exact Hamiltonian

for the wavefunctions of Laughlin (R.B. Laughlin, 1983b) in his model of the FQHE. The Hamiltonian derived will be shown to be a Hamiltonian of non-interacting electrons in contrast with the results of S.M. Girvin (S.M. Girvin, 1983).

2.1 THEORY OF 2D ELECTRONS

The Hamiltonian of a physical system reflects the symmetries and conservation laws of the system. Some of these symmetries can be used to derive exact relations between various physical quantities, or even to derive their functional form. For example the momentum or current conservation can be used to derive exact relations, the Ward-Takahashi relations, between the response functions of the system. The importance of these relations is that they can be used to ensure that later approximations of the response functions do also reflect the initial symmetries of the system.

2.1.1 The Model

A quasi-2D-electron gas is a system of 3D electrons which are free to move in two directions (the plane x_1, x_2) while their motion in the third direction (x_3) is restricted by a confining potential $V(x_3)$.

As was discussed in Chapter 1 many properties of the q-2D system are successfully described by a model of 2D electrons, however it will be shown later that when the interaction of a q-2D electron gas with dynamic 3D electromagnetic fields is treated, it may be necessary to consider the finite thickness of the system (the confining potential).

It will therefore be important to clearly distinguish between the 2D

and 3D variables. To do so the 2D co-ordinates will be denonated by:

$$\tilde{x} \equiv (x_1, x_2) \tag{2.1}$$

while the 3D co-ordinates are:

$$\bar{x} \equiv (x_1, x_2, x_3) = (\tilde{x}, x_3) \tag{2.2}$$

A corresponding notation will be used for the Fourier transformed co-ordinates. The full 3D Hamiltonian of the q-2D electron gas before the "dimensional reduction" is:

$$H = H_o + H_{Coul} + H_{imp} + H_{spin} + H_{confining} \tag{2.3}$$

with:

$$H_o = \frac{1}{2m}\int d\bar{x}\hat{\bar{\psi}}_s^{+}(\bar{x}t)\left[-i\hbar\bar{\nabla} + \frac{e}{c}\bar{A}_{ext}(\bar{x}) + \frac{e}{c}\bar{A}(\bar{x}t)\right]^2 \hat{\bar{\psi}}_s(\bar{x}t) \tag{2.4a}$$

where $\bar{A}_{ext}$ and $\bar{A}$ are c-number vector fields but the field operators satisfy the usual anticommutation relation:

$$\{\hat{\bar{\psi}}_s(\bar{x}t),\ \hat{\bar{\psi}}_r^{+}(\bar{x}'t)\} = \delta_{s,r}\,\delta(\bar{x} - \bar{x}') \tag{2.4b}$$

m is the effective electron mass, -e is the electronic charge, c is the speed of light and s is a spin index of the wave operator. (The Einstein summation convention is assumed).

$\bar{A}_{ext}$ is the external c-number vector potential that leads to the constant magnetic field B in the x_3 direction. $\bar{A}$ represents the internal vector potential, in which case its dynamics has to be included also. Since the vector field is not considered to be an operator, its dynamics will be described by its wave equation. Both these vector fields will be

considered using the Coloumb gauge:

$$\bar{\nabla}\cdot\bar{A} = 0 \tag{2.5}$$

H_{Coul} represents the Coulomb interaction in between the electrons:

$$H_{coul} = \frac{1}{2}\int d\bar{x}d\bar{x}'\ \hat{\Psi}_s^+(\bar{x}t)\ \hat{\Psi}_r^+(\bar{x}'t)\ U(\bar{x}-\bar{x}')\ \hat{\Psi}_r(\bar{x}'t)\ \hat{\Psi}_s(\bar{x}t) \tag{2.6}$$

where U is the Coulomb potential.

The interaction of the electrons with impurities in the system is described by:

$$H_{imp} = \sum_i \sum_\mu \int d\bar{x}\ \hat{\Psi}_s^+(\bar{x}t)\ \hat{\Psi}_s(\bar{x}t)\ V_\mu(\bar{x}-\bar{x}_i) \tag{2.7}$$

Here $V_\mu(\bar{x}-\bar{x}_i)$ is the potential of the μ-type impurity located at the site $\bar{x}_i$.

The interaction between the spin and the magnetic field has the Hamiltonian:

$$H_{spin} = \frac{1}{2}g\mu_B \sum_{i=1}^{3} \int d\bar{x}\ \hat{\Psi}_s^+(\bar{x}t)\ \left[\bar{\nabla}x(\bar{A}_{ext}(\bar{x}) + \bar{A}(\bar{x}t))\right]_i$$

$$(\sigma_i)_{sr}\ \hat{\Psi}_r(\bar{x}t) \tag{2.8}$$

where g is the electron g-factor, μ_B is the Bohr magneton and $(\sigma_i)_{sr}$ is the s,r element of the i^{th} component of the Pauli spin matrix.

The reduction of an effective 2D Hamiltonian will be accomplished by separating the wave-operator into parts:

$$\hat{\Psi}_s^+(\bar{x}t) \equiv \sum_n \hat{\phi}_s^+(\tilde{x}t)\ \chi_n(x_3) \tag{2.9}$$

where $\hat{\phi}_s^+(xt)$ is the 2D wave operator of the electrons in the free plane obeying the usual equal time fermionic anticommutation relation:

$$\{\hat{\phi}_s(\tilde{x}t), \hat{\phi}_r^+(\tilde{x}'t)\} = \delta_{s,r}\,\delta(\tilde{x} - \tilde{x}') \tag{2.10}$$

$\chi_n(x_3)$ on the other hand are the first quantized wavefunctions found from the one dimensional Schrödinger equation with the confining potential. Later on (2.9) will be truncated so that only the lowest state $\chi \equiv \chi_0$ is considered since in many experiments only the lowest subband is kept occupied.

This dimensional reduction is not trivial in all cases, and will be performed for each calculation taking into account the dynamics of the electromagnetic vector potentials.

Some calculations will be performed at zero temperature (T = 0), the groundstate $|\Phi_0\rangle$ of the noninteracting electrons will then be chosen in relation with the external constant magnetic field in the x_3 direction. In case of no magnetic field the usual fermi sea will be employed, while a certain number of Landau levels will be used in the case of an external field. The Heisenberg groundstate is denoted by $|\Psi_0\rangle$.

The density and the current density for the Hamiltonian (2.3) are:

$$\hat{\rho}(\bar{x}t) \equiv -e\hat{\Psi}_s^+(\bar{x}t)\,\hat{\Psi}_s(\bar{x}t) \tag{2.11a}$$

$$\begin{aligned}\hat{J}_k(\bar{x}t) \equiv{}& \frac{ie\hbar}{2m}\left[\hat{\Psi}_s^+(\bar{x}t)\,\partial_k\hat{\Psi}_s(\bar{x}t) - (\partial_k\hat{\Psi}_s^+(\bar{x}t))\,\hat{\Psi}_s(\bar{x}t)\right] \\ &- \frac{e^2}{mc}\left[\bar{A}_{ext}(\bar{x}) + \bar{A}(\bar{x}t)\right]_k \hat{\Psi}_s^+(\bar{x}t)\,\hat{\Psi}_s(\bar{x}t) \\ &- \frac{g\mu_B c}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\varepsilon_{kji}\,\partial_j\left[\hat{\Psi}_s^+(\bar{x}t)\,(\sigma_i)_{sr}\,\hat{\Psi}_r(\bar{x}t)\right]\end{aligned} \tag{2.11b}$$

ε_{kji} is the antisymmetric levi-Civita tensor. The last term of the current density, the spin current, has to be derived using the Nother's theorem.

In the work to follow the vector potential A will be considered to be a classical c-number field.

2.1.2 Response Functions

The response of a system, to any outside field or a probe, can be calculated in terms of the so-called response functions or retarded correlation functions (see section 2.2.2) (A. Isihara, 1971) (A.A. Abrikosov, 1963) (A.L. Fetter, 1971) (E.M. Lifshitz, 1980).

The density-density response function is defined as:

$$iD^R_{oo}(\bar{x}t, \bar{x}'t') \equiv \theta(t - t')\langle\Psi_o|[\hat{\rho}(\bar{x}t), \hat{\rho}(\bar{x}t)]|\Psi_o\rangle \qquad (2.12)$$

and the current-current response function:

$$iD^R_{ij}(\bar{x}t, \bar{x}'t') \equiv \theta(t - t')\langle\Psi_o|[\hat{J}_i(\bar{x}t), \hat{J}_j(\bar{x}'t')]|\Psi_o\rangle \qquad (2.13)$$

θ is here the Heaviside unit step function:

$$\theta(x) \equiv \frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{e^{i\omega x}}{\omega - i\varepsilon}\,d\omega = \begin{cases} 1 & \text{if } x>0 \\ 1/2 & \text{if } x=0 \\ 0 & \text{if } x<0 \end{cases} \qquad (2.14)$$

and ε is a vanishingly small positive number (B.W. Roos, 1969). It is also possible to define the current-density response function:

$$iD_{io}(\bar{x}t, \bar{x}'t') \equiv \theta(t - t')\langle\Psi_o|[\hat{J}_i(\bar{x}\ t\ ), \hat{\rho}(\bar{x}'t')]|\Psi_o\rangle \qquad (2.15)$$

These response functions are connected by the Ward-Takahashi relation

(H.S. Green, 1953) (K. Nishijima, 1960) (Y. Takahashi, 1957,1977a,b, 1978) (it is not essential here whether one is dealing with the 3D response functions or the 2D counterparts);

$$i\partial_t D^R_{oo}(\bar{x}t, \bar{x}'t') + i\partial D^R_{io}(\bar{x}t, \bar{x}'t') =$$

$$\delta(t - t')\langle\Psi_o|[\hat{\rho}(\bar{x}t), \hat{\rho}(\bar{x}'t')]|\Psi_o\rangle$$

$$+ \theta(t - t)\langle\Psi_o|[\partial_t\hat{\rho}(\bar{x}t) + \partial_i\hat{J}_i(\bar{x}t), \hat{\rho}(\bar{x}'t')]|\Psi_o\rangle = 0 \quad (2.16)$$

where the Einstein summation convention is used for the spatial part of the inner product.

The second equality holds since the equal time commutator of the density operates does vanish, and the current is assumed to be conserved:

$$\partial_t\hat{\rho}(\bar{x}t) + \partial_i\hat{J}_i(\bar{x}t) = 0 \quad (2.17)$$

The W-T relation therefore reflects the current conservation:

$$i\partial_t D^R_{oo}(\bar{x}t,\bar{x}'t') + i\partial_i D^R_{io}(\bar{x}t, \bar{x}'t') = 0 \quad (2.18)$$

If the system is time translationally invariant then the response functions depend on the time-difference t - t', so the following Fourier transform can be defined:

$$D^R_{oo}(\bar{x},\bar{x}',\omega) = \int_{-\infty}^{\infty} d(t - t')e^{i(t - t')\omega} D^R_{oo}(\bar{x},\bar{x}',t - t') \quad (2.19)$$

And the W-T relation becomes:

$$-i\omega D^R_{oo}(\bar{x},\bar{x}',\omega) + \partial_i D^R_{io}(\bar{x},\bar{x}',\omega) = 0 \quad (2.20)$$

This relation (2.20) can now be used to derive the so called f-sum rule, an exact relation. Integration over ω in (2.20) leads to:

$$i \int_{-\infty}^{\infty} \omega d\omega \; D^R_{oo}(\bar{x},\bar{x}',\omega) = \partial_i \int_{-\infty}^{\infty} d\omega \; D^R_{io}(\bar{x},\bar{x}',\omega)$$

$$= 2\pi\partial_i \; D^R_{io}(\bar{x},\bar{x}',\; t - t' = 0) \tag{2.21}$$

To evaluate the last term of (2.21) the following commutator, of the full current $\hat{J}_i$ (2.11b) and the density operator $\hat{\rho}$ (2.11a), is needed:

$$[\hat{J}_i(\bar{x}t),\; \hat{\rho}(\bar{x}'t)] = -\frac{ie^2\hbar}{m} \hat{\rho}(\bar{x}t)\partial_i \; \delta(\bar{x} - \bar{x}') \tag{2.22}$$

Using this expression (2.22) in (2.21) gives:

$$i \int_{-\infty}^{\infty} \omega d\omega \; D^R_{oo}(\bar{x},\bar{x}',\omega) = -\frac{\pi\hbar e^2}{m}\partial_i \; n_e(\bar{x})\partial_i \; \delta(\bar{x} - \bar{x}') \tag{2.23}$$

where:

$$n_e(\bar{x}) \equiv \langle\Psi_o|\hat{\rho}(\bar{x})|\Psi_o\rangle \tag{2.24}$$

$D^R_{oo}(\bar{x}t,\bar{x}'t')$ is a real function as can be seen from the definition (2.12), therefore the imaginary part of its Fourier transform has to be anti-symmetric:

$$\text{Im } D^R_{oo}(\bar{x},\bar{x}',-\omega) = -\text{Im } D^R_{oo}(\bar{x},\bar{x}',\omega) \tag{2.25}$$

combining equations (2.24) and (2.23) then leads to the f-sume rule:

$$\int_0^{\infty} \omega d\omega \; \text{Im } D^R_{oo}(\bar{x},\bar{x}',\omega) = \frac{\pi\hbar e^2}{2m} \partial_i n_e(\bar{x})\partial_i \; \delta(\bar{x} - \bar{x}') \tag{2.26}$$

The f-sume rule (2.26) is valid for a system in a magnetic field (no translational invariance) and with a groundstate of non-uniform density.

If the system has a uniform density and is translationally invariant, then eq. (2.25) takes the familiar form (A.A. Abrikosov, 1963) (D. Pines, 1971):

$$\int_0^\infty \omega d\omega \ \mathrm{Im}\ D^R_{oo}(\bar{q},\omega) = -\frac{\pi\hbar n_e e^2}{2m} q^2 \qquad (2.27)$$

where the following spatial Fourier transform has been introduced:

$$D^R_{oo}(\bar{q},\omega) = \int_{-\infty}^{\infty} d\bar{x}\ e^{-i\bar{x}\cdot\bar{q}}\ D^R_{oo}(\bar{x},\omega) \qquad (2.28)$$

There is another important relation between the current and density response functions, as will be seen later it relates the longitudinal part of the current response to the density response.

$$i\partial_i\partial'_j\ D^R_{ij}(\bar{x}t,\bar{x}'t') = i\partial_t\partial_{t'}\ D^R_{oo}(\bar{x}t,\bar{x}'t') + \frac{ie^2 n_e \hbar}{m}$$

$$\delta(t-t')\ \nabla^2\delta(\bar{x}-\bar{x}') \qquad (2.29)$$

or in Fourier space if $D^R_{ij}(\bar{x}t,\bar{x}'t') = D^R_{ij}(\bar{x}-\bar{x}',t-t')$:

$$k_i k_j\ D^R_{ij}(\bar{k},\omega) = \omega^2\ D^R_{oo}(\bar{k},\omega) - \frac{e^2 n_e \hbar}{m} k^2 \qquad (2.30)$$

This relation is also derived by using the current conservation (the eq. of continuity) and the commutation relation (2.22).

2.2 ELECTROMAGNETIC PROPERTIES OF 2D SYSTEMS

The transverse current response functions have not been calculated before for the q-2D electron gas. In this section these functions will be calculated and used in conjunction with self consistent linear response methods to study the propagation of electromagnetic waves or the transverse plasmon in 2D electron systems.

The cases of external or no external magnetic field have to be considered separately, since the separation into transverse and longitudinal

parts is complicated by the external field.

It is also of fundamental importance to consider properly the finite thickness of the q-2D electron gas systems, due to the 3D nature of the electromagnetic field.

2.2.1 The Model (B = 0)

The electrons will be considered to be free to move in the plane while they are in the ground state $\chi(x_3)$ of the confining potential $V(x_3)$ with respect to the motion along the x_3 direction. Only the interaction of the electrons via the vector potential $\bar{A}$ will be taken into account, since the Coulomb-interaction has no effects on the transverse properties of the system.

The Hamiltonian of the system is:

$$H = \int d\tilde{x}\, \hat{\phi}_s^+(\tilde{x}t)\left[-\frac{\hbar^2}{2m}\tilde{\nabla}^2 + E_3\right]\hat{\phi}_s(\tilde{x}t) + H_{int} \qquad (2.31)$$

Here the separation of the 3D wave operator (2.9) (2.10) has been performed and the groundstate energy E_3 for the wavefunction $\chi(x_3)$ is determined by the one dimensional Schrödinger eq.:

$$\left[-\frac{\hbar^2}{2m}\partial_3^2 + V(x_3)\right]\chi(x_3) = E_3\,\chi(x_3) \qquad (2.32)$$

Since $\chi(x_3)$ represents a boundstate the following conditions have to be satisfied:

$$\chi(x_3) \xrightarrow[x_3 \to \pm\infty]{} 0, \quad \int_{-\infty}^{\infty} dx_3 |\chi|^2 = 1 \qquad (2.33)$$

The interaction Hamiltonian H_{int} describes the interaction of the 2D

electrons via the vector field $\bar{A}$, and will be discussed later.

The components of the 3D free electron current lying in the $(x_1 - x_2)$-plane can be factorized as:

$$\hat{J}^0_\nu(\tilde{x},x_3,t) = |\chi(x_3)|^2 \frac{ie\hbar}{2m}\left[\hat{\phi}^+_s(\tilde{x}t)\partial_\nu\hat{\phi}_s(\tilde{x}t) - (\partial_\nu\hat{\phi}^+_s(\tilde{x}t))\hat{\phi}_s(\tilde{x}t)\right]$$
$$= |\chi(x_3)|^2 \hat{J}^0_\nu(\tilde{x},t), \ (\nu = 1,2) \qquad (2.34)$$

Greek indexes will be used to denote the first two components of any vector since with lowest boundstate wavefunction $\chi(x_3)$ can be assumed to be real, the third current component vanishes.

$$\hat{J}^0_3(\tilde{x},x_3,t) = \hat{\phi}^+_s(\tilde{x}t)\hat{\phi}_s(\tilde{x}t)\frac{ie\hbar}{2m}\left[\chi^*\partial_3\chi - (\partial_3\chi)^*\chi\right] = 0 \qquad (2.35)$$

As there is no external magnetic field $\bar{B}$, the 2D wave operator can be expanded in terms of plane waves and creation and annihilation operators:

$$\hat{\phi}_s(\tilde{x}t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\tilde{k}\cdot\tilde{x} - i\omega_k t}\hat{c}_s(\tilde{k}) \qquad (2.36)$$

where:

$$\hbar\omega_k = \frac{\hbar^2}{2m}\tilde{k}^2 + E_3 \qquad (2.37)$$

and the dot stands for the 2D inner product.

From (2.10) it can be found that the operators satisfy the usual anti-commutation relation:

$$\{\hat{c}_s(\tilde{k}), \hat{c}^+_r(\tilde{q})\} = \delta_{s,r}\,\delta(\tilde{k} - \tilde{q}) \qquad (2.38)$$

For the wavefunction $\chi(x_3)$, it will be convenient to introduce the

following Fourier transforms:

$$|\chi(x_3)|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk_3\, e^{ik_3x_3}\,\rho(k_3) \tag{2.39}$$

By use of (2.36) the 2D current can now be written as:

$$\hat{j}^o_\nu(xt) = \frac{1}{(2\pi)^2}\int d\tilde{k}\, e^{i\tilde{k}\cdot\tilde{x}}\left[-\frac{e}{2m}\int d\tilde{p}\, e^{-i(\omega_{\tilde{p}} + \hat{k} - \omega_k)t}\cdot(k_\nu + 2p_\nu)\quad \hat{c}^+_s(\tilde{p})\hat{c}_s(\tilde{p}+\tilde{k})\right]$$

$$\equiv \frac{1}{(2\pi)^2}\int d\tilde{k}\, e^{i\tilde{k}\cdot\tilde{x}}\,\hat{j}^o_\nu(\tilde{k}t) \tag{2.40}$$

The current operator (2.34) can be split into its transverse and longitudinal parts:

$$\hat{j}^o_\nu(x,x_3,t) = \frac{1}{(2\pi)^3}\int d\tilde{k}dk_3\, e^{i(\tilde{k}\cdot\tilde{x} + k_3x_3)}\cdot\left[\pi^{3D}_{\nu\mu}(\tilde{k},k_3) + \mathbb{I}^{3D}_{\nu\mu}(\tilde{k},k_3)\right]\cdot\rho(k_3)\;\hat{j}^o_\mu(\tilde{k},t) \tag{2.41}$$

by the introduction of the projection operators:

$$\mathbb{I}^{3D}_{\nu\mu}(\tilde{k},k_3) \equiv \frac{k_\nu k_\mu}{|\tilde{k}|^2 + k_3^2}, \qquad \pi^{3D}_{\nu\mu}(\tilde{k},k_3) \equiv \delta_{\nu\mu} - \mathbb{I}^{3D}_{\nu\mu}(\tilde{k},k_3) \tag{2.42}$$

The Greek indeces are used for the first two components of any 2D or 3D vector and the Einstein summation convention is used with respect to repeated indeces. The 2D counter parts of these projection operates can be defined:

$$\mathbb{I}_{\mu\nu}(\tilde{k}) \equiv \frac{k_\mu k_\nu}{|\tilde{k}|^2} = \mathbb{I}^{3D}_{\mu\nu}(\tilde{k},k_3 = 0), \qquad \pi_{\mu\nu}(\tilde{k}) \equiv \delta_{\mu\nu} - \mathbb{I}_{\mu\nu}(\tilde{k}) \tag{2.43}$$

in order to split the 2D current into its transverse and longitudinal parts:

$$\tilde{j}^{o}_{\mu}(\tilde{k},t) = \pi_{\mu\nu}(\tilde{k})\ \tilde{j}^{o}_{\nu}(\tilde{k},t) + \mathbb{L}_{\mu\nu}(\tilde{k})\ \tilde{j}^{o}_{\nu}(\tilde{k},t)$$

$$\equiv \tilde{j}^{ot}_{\mu}(\tilde{k},t) + \tilde{j}^{o\ell}_{\mu}(\tilde{k},t) \qquad (2.44)$$

The projection operators satisfy some important identities, valid for any 2D vector field $A_\lambda(\tilde{k})$ ($A_3 = 0$):

$$\mathbb{L}^{3D}_{\mu\nu}(\tilde{k},k_3)\ \mathbb{L}_{\nu\lambda}(\tilde{k})\ A_\lambda(\tilde{k}) = \mathbb{L}^{3D}_{\mu\lambda}(\tilde{k},k_3)\ A_\lambda(\tilde{k})$$

$$\mathbb{L}^{3D}_{\mu\nu}\,\pi_{\nu\lambda}\,A_\lambda = 0$$

$$\pi^{3D}_{\mu\nu}\,\mathbb{L}_{\nu\lambda}A_\lambda = (\mathbb{L}_{\mu\lambda} - \mathbb{L}^{3D}_{\mu\lambda})A_\lambda$$

$$\pi^{3D}_{\mu\nu}\,\mathbb{T}_{\nu\lambda}A_\lambda = \mathbb{T}_{\mu\lambda}A_\lambda \qquad (2.45)$$

The factorization of the 3D current now opens the possibility of writing the 3D current response function for a non-interacting electron gas in terms of the 2D response function:

$$D^{0,3D}_{\mu\nu}(\bar{x}t,\bar{x}'t') = |\chi(x_3)|^2\ |\chi(x_3)|^2\ D^{0}_{\mu\nu}(\tilde{x}t,\tilde{x}'t') \qquad (2.46a)$$

where:

$$iD^{0}_{\mu\nu}(\tilde{x}t,\tilde{x}'t') \equiv \theta(t-t')\langle\phi_0|[\hat{j}^{o}_{\mu}(\tilde{x}t),\ \hat{j}^{o}_{\nu}(\tilde{x}'t')]|\phi_0\rangle \qquad (2.46b)$$

Here the superscript R used to signify the retarded correlation in (2.12) (2.13) has been dropped for clarify of notation, and $|\phi_0\rangle$ stands for the non-interacting Fermi groundstate.

The splitting of the current (2.44) furthermore suggests the defini-

tion of the various response functions:

$$iD^{ot}_{\mu\nu}(\tilde{x}t,\tilde{x}'t') = \theta(t - t')\langle\phi_o|[\hat{j}^{ot}_{\mu}(\tilde{x}t), \hat{j}^{ot}_{\nu}(\tilde{x}'t')]|\phi_o\rangle \tag{2.47}$$

$$iD^{o\ell}_{\mu\nu}(\tilde{x}t,\tilde{x}'t') = \theta(t - t')\langle\phi_o|[\hat{j}^{o}_{\mu}(\tilde{x}t), \hat{j}^{o}_{\nu}(\tilde{x}'t')]|\phi_o\rangle \tag{2.48}$$

$$iD^{ot}_{\mu\nu}(\tilde{x}t,\tilde{x}'t') = \theta(t - t')\langle\phi_o|[\hat{j}^{ot}_{\mu}(\tilde{x}t), \hat{j}^{o}_{\nu}(\tilde{x}'t')]|\phi_o\rangle \tag{2.49}$$

$$iD^{o\ell t}_{\mu\nu}(\tilde{x}t,\tilde{x}'t') = \theta(t - t')\langle\phi_o|[\hat{j}^{o\ell}_{\mu}(\tilde{x}t), \hat{j}^{ot}_{\nu}(\tilde{x}'t')]|\phi_o\rangle \tag{2.50}$$

which are useful to express the current response function as:

$$D^{o}_{\mu\nu}(\tilde{x}t,\tilde{x}'t') = D^{ot}_{\mu\nu} + D^{o\ell}_{\mu\nu} + D^{ot}_{\mu\nu} + D^{o\ell t}_{\mu\nu} \tag{2.51}$$

later it will be shown that the mixed response functions $D^{ot\ell}_{\mu\nu}$ and $D^{o\ell t}_{\mu\nu}$ vanish, while the projection operators may be factorized in the Fourier transform of the remaining two:

$$\begin{aligned} D^{ot}_{\mu\nu}(\tilde{k},\omega) &= \mathbb{T}_{\mu\nu}(\tilde{k})\, D^{ot}(\tilde{k},\omega) \\ D^{o}_{\mu\nu}(\tilde{k},\omega) &= \mathbb{L}_{\mu\nu}(\tilde{k})\, D^{o}(\tilde{k},\omega) \end{aligned} \tag{2.52}$$

2.2.2 Self-Consistent Linear Response

The Hamiltonian for the interaction H_{int} (2.31) of the q-2D electrons with the vector field $\bar{A}$ is:

$$H_{int} = -\frac{1}{c}\int d\bar{x}\, A_{\nu}(\bar{x}t)\, \hat{J}_{\nu}(\bar{x}t) - \frac{e^2}{2mc^2}\int d\bar{x}\, A^2_{\nu}\, \hat{\psi}^{+}_{s}\hat{\psi}_{s} \tag{2.53}$$

where the current is now the full current for the total Hamiltonian (2.31):

$$\hat{J}_\nu(\bar{x}t) \equiv |\chi(x_3)|^2 \left[\hat{j}^0_\nu(\tilde{x}t) - \frac{e^2}{mc} A_\nu(\bar{x}t)\ \hat{\phi}^+_s(\tilde{x}t)\ \hat{\phi}_s(\tilde{x}t) \right] \qquad (2.54)$$

since the electrons are confined to the $(x_1 - x_2)$-plane one assumes that $A_3 = 0$, and therefore:

$$\hat{J}_3(x,t) = 0 \qquad (2.55)$$

According to linear response theory the time evolution of the expectation value of the current operator, to the lowest order in H_{int} is:

$$\langle \hat{J}_\mu(\bar{x}t) \rangle = - \frac{e^2 n_e}{mc} |\chi(x_3)|^2 A_\mu(\bar{x}t)$$

$$- \frac{1}{\hbar c} |\chi(x_3)|^2 \int_{-\infty}^{\infty} dt' d\tilde{x}'\ D^0_{\mu\nu}(\tilde{x} - \tilde{x}', t - t')$$

$$\int_{-\infty}^{\infty} dx'_3\ |\chi(x'_3)|^2 A_\nu(\bar{x}'t') \qquad (2.56)$$

where $D^0_{\mu\nu}$ is the 2D current response function defined in (2.46b), and no initial current was assumed. The expression (2.56) gives the expectation value of the induced current for an external c-number vector field A_ν. If on the other hand one is interested in finding the transverse plasmon of the system (the collective mode of the electrons mediated by the A_ν-field), then the vector field should be the internal one created by the currents in the system:

$$\left[\bar{\nabla}^2 - \frac{1}{c^2} \partial_t^2 \right] A_\nu(\bar{x},t) = - \frac{4\pi}{c} \langle \hat{J}^t_\nu(\bar{x},t) \rangle \qquad (2.57)$$

where $\langle J^t_\nu(\bar{x}t) \rangle$ is the 3D transverse part of $\langle \hat{J}_\nu \rangle$; it is also assumed here that there are no external currents.

To obtain the transverse plasmon it is now possible to use eq. (2.57)

to eliminate self-consistently the vector field A_ν from eq. (2.56), and thereby get an equation for the expectation value of the induced current. The nontrivial solution of that expression would correspond to the transverse plasmon. It is possible to use eq. (2.57) to eliminate the current from eq. (2.56) and thereby obtain an expression for the propagation of the vector field in the 2D electron system. This indicates that the transverse plasmon is nothing else but the propagation of a transverse electromagnetic wave in the system.

The elimination of $\bar{A}$ from eq. (2.56) is best carried out in k-space, so the following Fourier transforms are introduced:

$$\langle \hat{J}_\mu(\bar{x},t)\rangle = \frac{1}{(2\pi)^4}\int d\bar{k}d\omega\, e^{i\bar{k}\cdot\bar{x} - i\omega t}\, J_\mu(\tilde{k},k_3,\omega) \tag{2.58}$$

$$A_\mu(\bar{x},t) = \frac{1}{(2\pi)^4}\int d\bar{k}d\omega\, e^{i\bar{k}\cdot\bar{x} - i\omega t}\, A_\mu(\tilde{k},k_3,\omega) \tag{2.59}$$

The retarded solution of the wave equation (2.57) for the electron system placed in a vacuum is given by:

$$A_\mu(k,k_3,\omega) = \frac{4\pi}{c}\, \hat{J}^t_\mu(\tilde{k},k_3,\omega)\, \Delta(\tilde{k},k_3,\omega) \tag{2.60}$$

with:

$$\left[\Delta(k,k_3,\omega)\right]^{-1} = |\tilde{k}|^2 + k_3^2 - \frac{1}{c}(\omega + i\varepsilon)^2 \tag{2.61}$$

where ε is a vanishingly small positive number, and

$$J^t_\mu(k,k_3,\omega) \equiv \pi^{3D}_{\mu\lambda}(\tilde{k},k_3)\, J_\lambda(\tilde{k},k_3,\omega) \tag{2.62}$$

The Fourier transform of the linear response expression (2.56) becomes:

$$J_\mu(\tilde{k},k_3,\omega) = -\frac{e^2 n_e}{2\pi mc}\int dq_3\ \rho(k_3 - q_3)\ A_\mu(\tilde{k},q_3,\omega)$$

$$-\frac{1}{2\pi\hbar c}\ \rho(k_3)\ D^0_{\mu\nu}(\tilde{k},\omega)\int dq_3\ \rho(-q_3)\ A_\nu(\tilde{k},q_3,\omega) \qquad (2.63)$$

where definitions (2.58), (2.59) and (2.39) have been used. The vector field A_ν can now be eliminated from (2.63) by using (2.60). First, for simplification of notation define:

$$\Omega_{\mu\lambda}(\tilde{k},k_3,\omega) \equiv \frac{4\pi}{c}\,\Delta(\tilde{k},k_3,\omega)\,\pi^{3D}_{\mu\lambda}(\tilde{k},k_3) \qquad (2.64)$$

Then (2.63) becomes:

$$J_\mu(\tilde{k},k_3,\omega) = -\frac{e^2 n_e}{2\pi mc}\int dq_3\ \rho(k_3 - q_3)\ \Omega_{\mu\nu}(\tilde{k},q_3,\omega)\ J_\nu(\tilde{k},q_3,\omega) \qquad (2.65)$$

$$-\frac{1}{2\pi\hbar c}\ \rho(k_3)D^0_{\mu\lambda}(\tilde{k},\omega)\int dq_3\ \rho(-q_3)\ \Omega_{\lambda\nu}(\tilde{k},q_3,\omega)\ J_\nu(\tilde{k},q_3,\omega)$$

which can be written compactly as:

$$\int dq_3\ K_{\mu\nu}(\tilde{k},k_3,\omega)\ J_\nu(\tilde{k},q_3,\omega) = 0 \qquad (2.66)$$

with:

$$K_{\mu\nu}(\tilde{k},k_3,\omega) \equiv \delta_{\mu\nu}\ \delta(q_3 - k_3) + \frac{e^2 n_e}{2\pi mc}\ \rho(k_3 - q_3)\ \Omega_{\mu\nu}(\tilde{k},q_3,\omega)$$

$$+\frac{1}{2\pi\hbar c}\ \rho(k_3)D^0_{\mu\lambda}(\tilde{k},\omega)\ \rho(-q_3)\ \Omega_{\lambda\nu}(\tilde{k},q_3,\omega) \qquad (2.67)$$

From (2.66) it is possible to find whether a transverse plasmon mode exists in the system or not. If a relation, $\omega = \omega(k)$ can be found for $K_{\mu\nu}$ such that a non-vanishing J_ν satisfies (2.66), then this expression $\omega = \omega(k)$ is the dispersion relation for a transverse plasmon in the system.

In order to accomplish this the following assumption has to be made:

$$J_\mu(\tilde{k},k_3,\omega) = \rho(k_3)\langle j_\mu(\tilde{k},\omega)\rangle \qquad (2.68)$$

This factorization holds exactly for the current of the free electrons (2.34), while for the current of the interacting electrons (2.54) it is an approximation, the validity of which depends on the form of $\rho(k_3)$.

With the definition:

$$\Pi_{\mu\nu}(\tilde{k},k_3,\omega) \equiv \int dq_3\ \rho(k_3 - q_3)\ \rho(q_3)\ \Omega_{\mu\nu}(\tilde{k},q_3,\omega) \qquad (2.69)$$

together with (2.68), the expression (2.63) becomes:

$$\rho(k_3)\langle j_\mu(k,\omega)\rangle = -\frac{e^2 n_e}{2\pi mc}\,\Pi_{\mu\nu}(\tilde{k},k_3,\omega)\langle j_\nu(\tilde{k},\omega)\rangle$$

$$-\frac{1}{2\pi\hbar c}\,\rho(k_3)D^0_{\mu\lambda}(\tilde{k},\omega)\ \Pi_{\lambda\nu}(\tilde{k},0,\omega)\langle j_\nu(\tilde{k},\omega)\rangle \qquad (2.70)$$

If the transverse wave is to travel in the system of electrons along the $(x_1 - x_2)$-plane then k_3 should be set equal to zero in eq. (2.70). This together with the definition of the projection operators (2.42) and (2.43) leads to:

$$\Pi_{\mu\nu}(\tilde{k},0,\omega)\langle j_\nu(\tilde{k},\omega)\rangle = \frac{4\pi}{c}\int dq_3\ \rho(-q_3)\ \rho(q_3)\ \ \Delta(\tilde{k},q_3,\omega)\ \pi_{\mu\nu}(\tilde{k})\cdot$$

$$\langle j_\nu(\tilde{k},\omega)\rangle$$

$$= \Gamma(\tilde{k},\omega)\langle j^t_\nu(\tilde{k},\omega)\rangle \qquad (2.71)$$

where:

$$\Gamma(\tilde{k},\omega) \equiv \frac{4\pi}{c}\int dq_3\ \rho(-q_3)\ \rho(q_3)\ \Delta(\tilde{k},q_3,\omega) \qquad (2.72)$$

The normalization condition for $\chi(x_3)$ (2.33) is equivalent to stating that:

$$\rho(k_3 = 0) = 1 \tag{2.73}$$

Using (2.71), (2.73) and

$$\Pi_{\mu\nu}(\tilde{k})\, D^{0}_{\nu\lambda}(\tilde{k},\omega) = D^{0t}_{\mu\lambda}(\tilde{k},\omega) \tag{2.74}$$

(see (2.52)) allows (2.70) to be written as:

$$\langle j^{t}_{\mu}(\tilde{k},\omega)\rangle = -\frac{e^2 n_e}{2\pi mc}\,\Gamma(\tilde{k},\omega)\langle j^{t}_{\mu}(\tilde{k},\omega)\rangle$$

$$-\frac{1}{2\pi\hbar c}\,\Gamma(\tilde{k},\omega)\, D^{0t}_{\mu\nu}(\tilde{k},\omega)\langle j^{t}_{\mu}(\tilde{k},\omega)\rangle \tag{2.75}$$

If in addition one uses (2.52) then eq. (2.75) reduces to:

$$\left[1 + \left(\frac{e^2 n_e}{2\pi mc} + \frac{1}{2\pi\hbar c}\, D^{0t}(\tilde{k},\omega)\right)\Gamma(\tilde{k},\omega)\right]\langle j^{t}_{\nu}(\tilde{k},\omega)\rangle = 0 \tag{2.76}$$

From which it can be seen that the dispersion relation $\omega = \omega(k)$ is determined from the equation:

$$\frac{1}{\Gamma(\tilde{k},\omega)} + \frac{e^2 n_e}{2\pi mc} + \frac{1}{2\pi\hbar c}\; D^{0t}(\tilde{k},\omega) = 0 \tag{2.77}$$

In order to solve this equation, one needs to specify $\rho(k_3)$ (i.e. $\chi(x_3)$) to evaluate:

$$\Gamma(\tilde{k},\omega) = \frac{4\pi}{c}\int dq_3\; \rho(-q_3)\;\rho(q_3)\;\Delta(\tilde{k},q_3,\omega) \tag{2.78}$$

and the transverse current response $D^{0t}(\tilde{k},\omega)$ has to be calculated.

2.2.3 Evaluation of the Response Function

In this section the transverse response function will be evaluated

exactly, after which various physically interesting limiting cases will be presented. The results will also be given for the longitudinal response function.

The response function for the transverse current is given by:

$$iD^{ot}_{\mu\nu}(\tilde{x}t,\tilde{x}'t') = \theta(t - t')\langle\phi_o|[\hat{j}^{ot}_{\mu}(\tilde{x}t),\ \hat{j}^{ot}_{\nu}(\tilde{x}'t')]|\phi_o\rangle \qquad (2.79)$$

The commutator is evaluated using (2.40) and (2.38) and the remaining vacuum expectation values are of the form:

$$\langle\phi_o|\hat{c}^{+}_{s}(\tilde{p})\ \hat{c}_{r}(\tilde{k} + \tilde{q})|\phi_o\rangle = \delta_{sr}\ \delta(\tilde{p} - \tilde{k} - \tilde{q})\ \theta(k_F - p) \qquad (2.80)$$

where $|\phi_o\rangle$ is the vacuum of noninteraction fermions, and k_F is the fermi wave vector:

$$k_F^2 = 2\pi n_e \qquad (2.81)$$

in the case of two degrees of freedom for the spin. If the Fourier transform is then defined as:

$$D^{ot}_{\mu\nu}(\tilde{k},\omega) \equiv \int_{-\infty}^{\infty} d(\tilde{x} - \tilde{x}')\ d(t - t')\ e^{i(\tilde{x} - \tilde{x}')\cdot\tilde{k} - i(t - t)\omega}$$

$$\cdot\ D^{ot}_{\mu\nu}(\tilde{x} - \tilde{x}', t - t') \qquad (2.82)$$

for a spatially isotropic system, then the transverse current response function becomes:

$$D^{ot}_{\mu\nu}(\tilde{k},\omega) = \frac{1}{2\pi^2}\left(\frac{e\hbar}{m}\right)^2 \int_0^{k_F} pdp \int_0^{2\pi} d\theta\ \Pi_{\mu\lambda}(\tilde{k})p_{\lambda}\Pi_{\nu\delta}(\tilde{k})p_{\delta}$$

$$\cdot\left[\frac{1}{\omega - (\varepsilon_{\tilde{k}+\tilde{p}} - \varepsilon_{\tilde{p}}) + i\eta} - \frac{1}{\omega + (\varepsilon_{\tilde{k}+\tilde{p}} - \varepsilon_{\tilde{p}}) + i\eta}\right] \qquad (2.83)$$

where $\eta \to 0^+$ and

$$\varepsilon_{\tilde{p}} = \frac{\hbar}{2m}\tilde{p}^2 + E_3 \tag{2.84}$$

Without doing the angular integration its symmetry can be explored to find that the integral can be written in the much simpler form:

$$D^{ot}_{\mu\nu}(\tilde{k},\omega) = \frac{1}{2\pi^2}\left(\frac{e\hbar}{m}\right)^2 \pi_{\mu\nu}(\tilde{k}) \int_0^{k_F} pdp \int_0^{2\pi} d\theta p^2 \sin^2\theta$$

$$\cdot\left[\frac{1}{\omega - \frac{\hbar\tilde{k}\cdot\tilde{p}}{m} - \frac{\hbar\tilde{k}^2}{2m} + i\eta} - \frac{1}{\omega - \frac{\hbar\tilde{k}\cdot p}{m} + \frac{\hbar\tilde{k}^2}{2m} + i\eta}\right] \tag{2.85}$$

where the transverse operators have been simplified and factorized outside the integral, and eq. (2.84) has been used.

In order to evaluate the integral (2.85) it is convenient to define the dimensionless frequency and wave vector:

$$\nu \equiv \frac{m\omega}{\hbar k_F^2}, \quad u = \frac{|\tilde{k}|}{k_F} \tag{2.86}$$

leading to:

$$D^{ot}_{\mu\nu}(u,\nu) = \frac{1}{2\pi^2}\frac{e^2\hbar k_F^2}{m}\int_0^1 x^3 dx \int_{-\pi}^{\pi} d\theta \sin^2\theta$$

$$\cdot\left[\frac{1}{\nu - \frac{u^2}{2} + ux\cos\theta + i\eta} - \frac{1}{\nu + \frac{u^2}{2} + ux\cos\theta + i\eta}\right] \tag{2.87}$$

Now the imaginary and the real part can be separated using the identity:

$$\frac{1}{\omega \pm i\eta} = \mathcal{P}\frac{1}{\omega} \mp i\pi\delta(\omega) \tag{2.88}$$

where $\mathcal{P}$ stands for the principal value. The angular integral is then of the form:

$$\int_{-\pi}^{\pi} d\theta \, \frac{\sin^2\theta}{a + b\cos\theta} = \frac{2\pi}{b^2}\left[a - \sqrt{a^2 - b^2}\left(\theta(a) - \theta(-a)\right)\theta(a^2 - b^2)\right] \quad (2.89)$$

where a and b stand for the appropriate combination of u and ν needed in each case.

The rest of the integration is straight forward, and if one defines:

$$D^{ot}_{\mu\nu}(\tilde{k},\omega) = \Pi_{\mu\nu}(\tilde{k})\, D^{ot}(k,\omega) \equiv \frac{e^2 \hbar k_F^2}{\pi m}\Pi_{\mu\nu}(\tilde{k})\, \tilde{D}^{t}(k,\omega) \quad (2.90)$$

then the results are:

$$\mathrm{Re}\, \tilde{D}^{t}(k,\omega) = \frac{u^2}{12} + \left(\frac{\nu}{u}\right)^2 - \frac{1}{2} + g_-(u,\nu) - g_+(u,\nu) \quad (2.91)$$

and:

$$\mathrm{Im}\, \tilde{D}^{t}(k,\omega) = f_+(u,\nu) - f_-(u,\nu) \quad (2.92)$$

where:

$$g_\pm\ (u,\nu) = \frac{1}{3u}\,\mathrm{sgn}(v_\pm)\,\theta(v_\pm^2 - 1)\sqrt{(v_\pm^2 - 1)^3} \quad (2.93)$$

$$f_\pm\ (u,\nu) = \frac{1}{3u}\,\theta(1 - v_\pm^2)\sqrt{(1 - v_\pm^2)^3} \quad (2.94)$$

$$u = \frac{|k|}{k_F} = \frac{k}{k_F}, \quad \nu = \frac{m\omega}{\hbar k_F^2}, \quad v_\pm \equiv \frac{\nu}{u} \pm \frac{u}{2} \quad (2.95)$$

The longitudinal current response function (2.48) can be evaluated by similar methods as the transverse one, giving:

$$D^{o\ell}_{\mu\nu}(\tilde{k},\omega) = \frac{e^2\hbar k_F^2}{\pi m} \mathbb{L}_{\mu\nu}(\tilde{k})\, \tilde{D}^{\ell}(k,\omega) \tag{2.96}$$

where:

$$\text{Re}\, \tilde{D}^{\ell}(k,\omega) = \frac{\nu^2}{u^4}\left[-\frac{u^4}{2\nu^2} - u^2 - \tilde{g}_-(u,\nu) + \tilde{g}_+(u,\nu)\right] \tag{2.97}$$

and:

$$\text{Im}\, \tilde{D}^{\ell}(k,\omega) = \frac{\nu^2}{u^4}\left[\tilde{f}_+(u,\nu) - \tilde{f}_-(u,\nu)\right] \tag{2.98}$$

with:

$$\tilde{g}_{\pm}\ (u,\nu) \equiv \text{sgn}(\nu_{\pm})\ \theta(\nu_{\pm}^2 - 1)\ u\sqrt{\nu_{\pm}^2 - 1} \tag{2.99}$$

$$\tilde{f}_{\pm}\ (u,\nu) \equiv \theta(1 - \nu_{\pm}^2)\ u\sqrt{1 - \nu_{\pm}^2} \tag{2.100}$$

The symmetry with respect to the angular variable θ in the integral expressions for the mixed response functions $D^{ot\ell}_{\mu\nu}$ and $D^{o\ell t}_{\mu\nu}$ show that these vanish.

$$D^{ot\ell}_{\mu\nu} = D^{o\ell t}_{\mu\nu} = 0 \tag{2.101}$$

From (2.92) and (2.98) it can be seen that Im $\tilde{D}^{t}$ and Im$\tilde{D}^{\ell}$ are non-vanishing only in the region:

$$-1 < \frac{\nu}{u} \pm \frac{u}{2} < 1 \tag{2.102}$$

This is shown in Fig. 1, where the shadowed regions give vanishing Im D^{ℓ} and Im D^{t}. In Fig. 1, the slope of $\overline{OE}$ is given by:

$$\tan\phi = \frac{\sqrt{2\pi n_e}}{m}\hbar \tag{2.103}$$

which is smaller than the speed of light c for $n_e << 10^{20}$ cm^{-2}. The following limiting forms for the response functions are of physical importance:

$$\mathrm{Re}\,\tilde{D}^t(u,\nu) = \begin{cases} \frac{1}{8}\left(\frac{u}{\nu}\right)^2\left[1 + \frac{1}{2}\left(\frac{u}{\nu}\right)^2 + \ldots\right] & (\nu = \text{finite}, u \to 0) \quad (2.104) \\ -\frac{1}{2} + \frac{u^2}{12} & (\nu = 0, u < 2) \quad (2.105) \\ -\frac{1}{2} - \frac{u^2}{12}\left(1 - \frac{4}{u^2}\right)^{3/2} + \frac{u^2}{12} & (\nu = 0, u > 2) \quad (2.106) \end{cases}$$

$$\mathrm{Re}\,\tilde{D}^\ell(u,\nu) = \begin{cases} \frac{1}{8}\left(\frac{u}{\nu}\right)^2\left[3 + u^2 + \frac{5}{2}\left(\frac{u}{\nu}\right)^2 + \ldots\right] & (\nu = \text{finite}, u \to 0) \quad (2.107) \\ -\frac{1}{2} & (\nu = 0, u < 2) \quad (2.108) \\ -\frac{1}{2} & (\nu = 0, u > 2) \quad (2.109) \end{cases}$$

The static long wavelength limit for Re D^t (2.105) was derived by A.K. Rajagopal (A.K. Rajagopal, 1977) and is in agreement with eq. (2.105). The longitudinal part of the current response function (2.97) and (2.98) can be compared to the density response function D^o_{oo}, calculated by F. Stern (F. Stern, 1967), by using the relation (2.30). These two calculations are in complete agreement.

2.2.4 2D Transverse Plasmon, B = 0

In this section the dispersion for the electomagnetic wave propagating in the $(x_1 - x_2)$-plane will be calculated.

As was mentioned before, the wavefunction $\rho(k_3)$ has to be specified so that the dispersion can be evaluated from eq. (2.77). Three different cases will be considered, (i) Lorentzian $\rho(k_3)$, (ii) Gaussian $\rho(k_3)$ and (iii) the zero thickness limiting case.

(i) The Lorentzian $\rho(k_3)$ is:

$$\rho(k_3) = \frac{a^2}{k_3^2 + a^2} \tag{2.110}$$

implying:

$$|\chi(x_3)|^2 = \frac{a}{2} e^{-|x_3|a} \tag{2.111}$$

Of course, the above $\chi(x_3)$ should be understood as an approximate solution to (2.32) on account of the differentiability at $x_3 = 0$. The parameter a can be interpreted as the inverse thickness of the system. In seeking for a dispersion relation for the transverse plasmon in the system the case $\omega > ck$ will be considerèd, expecting a gap $\omega(k = o) \neq 0$. The function $\Gamma(k,\omega)$ (2.78) can now be evaluated for $\omega > ck$:

$$\Gamma(k,\omega) = \frac{4\pi}{c} \int_{-\infty}^{\infty} dk_3 \frac{a^4}{(k_3^2 + a^2)^2} \frac{1}{\left[k^2 + k_3^2 - \frac{1}{c^2}(\omega + i\varepsilon)^2\right]}$$

$$= -\frac{2\pi^2 a}{c(a^2 + \gamma^2)} \left[\left(1 + \frac{2a^2}{a^2 + \gamma^2}\right) - \frac{i2a^3}{\gamma(a^2 + \gamma^2)}\right] \tag{2.112}$$

where:

$$\gamma^2 \equiv \frac{\omega^2}{c^2} - k^2, \quad k \equiv |\vec{k}| \tag{2.113}$$

In order to get analytical results, $\Gamma(k,\omega)$ has to be expanded in powers of

$\frac{ck}{\omega}$:

$$\Gamma(k,\omega) = \Gamma_0(\omega) + \Gamma_1(\omega)\left(\frac{ck}{\omega}\right)^2 + O\left(\left(\frac{ck}{\omega}\right)^4\right) \tag{2.114}$$

If a is finite, the above Γ_0 and Γ_1 both have imaginary parts. Their explicit forms are:

$$\Gamma_0(\omega) = -\frac{2\pi^2 ac}{(a^2c^2+\omega^2)}\left[1 + \frac{2a^2c^2}{a^2c^2+\omega^2}\right] + \frac{i4\pi^2a^4c^4}{\omega(a^2c^2+\omega^2)^2} \tag{2.115}$$

and

$$\Gamma_1(\omega) = -\frac{2\pi^2 ac\omega^2}{(a^2c^2+\omega^2)^2}\left[1 + \frac{4a^2c^2}{a^2c^2+\omega^2}\right] + \frac{i4\pi^2a^4c^4}{(a^2c^2+\omega^2)^2} \cdot \left[\frac{1}{2\omega} + \frac{2\omega}{a^2c^2+\omega^2}\right] \tag{2.116}$$

Under the assumption that the wave function is well localized in the k_3-space, a must be very small. In the limit $a \to 0^+$ $\rho(k_3)$ approaches a δ-function:

$$\lim_{a\to 0} \rho(k_3) = \lim_{a\to 0} a\sqrt{\pi}\,\frac{a}{\sqrt{\pi}(k_3^2 + a^2)} = \sqrt{\pi}\, a\delta(k_3) \tag{2.117}$$

meaning that the system is uniform in the x_3 direction with an infinite thickness 1/a. For small a, the above Γ_0 and Γ_1 become:

$$\Gamma_0 = -\frac{2\pi^2 ac}{\omega^2} + O\left(\frac{a^2c^2}{\omega^2}\right) \tag{2.118}$$

$$\Gamma_1 = -\frac{2\pi^2 ac}{\omega^2} + O\left(\frac{a^2c^2}{\omega^2}\right) \tag{2.119}$$

The imaginary parts do not survive in this limit. Thus for the Lorentzian:

$$\Gamma(k,\omega) = -\frac{2\pi^2 ac}{\omega^2}\left[1 + \frac{c^2k^2}{\omega^2}\right] \tag{2.120}$$

In the limit $\omega > ck$, the transverse current response function (2.104) is:

$$D^{ot}(k,\omega) = \frac{e^2\hbar^3 k_F^4}{8\pi m^3 c^2}\,\frac{c^2k^2}{\omega^2} + O\left(\frac{c^4k^4}{\omega^4}\right) \tag{2.121}$$

and in this limit the imaginary part vanishes. Using (2.120) and (2.121), one can solve (2.77) for small c^2k^2/ω^2 to obtain the dispersion relation.

$$\omega^2 = \frac{2\pi e^2 n_e}{m}\left(\frac{a}{2}\right) + \left(1 + \frac{1}{4}\left(\frac{\hbar k_F}{mc}\right)^2\right) c^2k^2 \tag{2.122}$$

It is interesting to see that the quantum correction in (2.122), the third term, is independent of a.

(ii) Next a Gaussian $\rho(k_3)$ is assumed, which can also be characterized by the parameter a.

$$\rho(k_3) = e^{-k_3^2/2a^2} \tag{2.123}$$

The x_3-space wave function for this $\rho(k_3)$ is:

$$|x(x_3)|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk_3\, e^{-k_3^2/2a^2}\, e^{ik_3x_3} = \frac{a}{\sqrt{2\pi}}\, e^{-(a^2/2)x_3^2} \tag{2.124}$$

As in the Lorentzian case, first the limiting case for small a is considered, then:

$$\Gamma(k,\omega) = -\frac{4\pi\sqrt{\pi}\,ac}{\omega^2 - c^2k^2} + O\left(\left(\frac{ac}{\omega}\right)^3\right) \tag{2.125}$$

The dispersion becomes then (for $\omega >> ck$):

$$\omega^2 = \frac{2\pi e^2 n_e}{m}\left(\frac{a}{\sqrt{\pi}}\right) + \left(1 + \frac{1}{4}\left(\frac{\hbar k_F}{mc}\right)^2\right)c^2k^2 \qquad (2.126)$$

The only difference between the Lorentzian and the Gaussian case is therefore the numerical factor multiplying a in the gap term.

(iii) Now the opposite case will be investigated, i.e. when a becomes large and the thickness of the system is small. Assume:

$$|\chi(x_3)|^2 = \delta(x_3), \quad \Phi(k_3) = 1 \qquad (2.127)$$

Then $\Gamma(k,\omega)$ in (2.78) becomes:

$$\Gamma(k,\omega) = \begin{cases} i4\pi^2(\omega^2 - c^2k^2)^{-1/2} & (\omega>ck) \\ \\ 4\pi^2(c^2k^2 - \omega^2)^{-1/2} & (\omega<ck) \end{cases} \qquad (2.128)$$

In order to obtain $\omega = \omega(k)$ from (2.77) it is necessary to have real Γ. Therefore due to (2.128) the limit $\omega<ck$ is required. On the other hand $\mathrm{Im}D^{ot}$ vanishes only for $\omega>\hbar k_F k/m$ as is shown in (2.102). Therefore

$$\frac{\hbar k_F k}{m} < \omega < ck \qquad (2.129)$$

must be satisfied, implying that (2.104) must be used as an expansion in $\frac{u}{v} = \hbar k_F k/m\omega$, then (2.77) becomes:

$$\frac{\sqrt{c^2k^2 - \omega^2}}{4\pi^2} + \frac{e^2 n_e}{2\pi mc} + \frac{1}{2\pi\hbar c}\cdot\frac{e^2\hbar^3 k_F^4}{8\pi m^3 c^2}\frac{c^2k^2}{\omega^2} = 0 \qquad (2.130)$$

which obviously does not have any real $\omega = \omega(k)$ solution. Therefore one is led to the conclusion that in this limit there is no transverse plasmon

in the system.

The results for the transverse plasmon dispersion may be summarized as follows: the transverse plasmon in a 2D electron system of thickness $\sigma \ll 1/a$ has the dispersion relation:

$$\omega^2 = \frac{2\pi e^2 n_e}{m}\left(\frac{\kappa}{\sigma}\right) + \left(1 + \frac{1}{4}\left(\frac{\hbar k_F}{mc}\right)^2\right)c^2k^2 \qquad (2.131)$$

where σ is the thickness of the sytem and κ is a numerical factor depending on the wave function $\chi(x_3)$. The result is valid for large σ. In the limit $\sigma \to 0$ there exists no plasmon in the system.

This result can be compared with the transverse plasmon in 3D systems (H. Matsumoto, 1980):

$$\omega^2 = \frac{4\pi e^2 n_{3D}}{m} + \left(1 + \frac{1}{5}\left(\frac{\hbar k_F}{mc}\right)^2\right)c^2k^2 \qquad (2.132)$$

which is also derived by the same method (the self consistent linear response SCLR) in appendix A. In comparing (2.132) and (2.131) it becomes evident how important it is to account for the finite thickness of the q-2D electron gas, in order to derive the gap in (2.131). It is also important to compare (2.131) to the longitudinal plasmon in 2D systems (F. Stern, 1967):

$$\omega^2 = \frac{2\pi e^2 n_e}{m}k + \frac{3}{4}\left(\frac{\hbar k_F}{mc}\right)^2 c^2k^2 \qquad (2.133)$$

and the 3D longitudinal plasmon (D. Pines, 1971)

$$\omega^2 = \frac{4\pi e^2 n_{3D}}{m} + \frac{3}{5}\left(\frac{\hbar k_F}{mc}\right)^2 c^2k^2 \qquad (2.134)$$

The 2D transverse plasmon has a definite gap depending on thickness of the

system, in contrast with the gapless 2D longitudinal plasmon.

2.2.5 The Model in the Case of an External Magnetic Field (B≠0)

When a magnetic field B is applied perpendicular to the q-2D electron gas, the free motion of the electrons along the $(x_1 - x_2)$-plane is quantized into Landau levels. In this and the next two sections the electromagnetic properties of such a system will be explored with respect to the current response function and the possible propagation of a transverse electromagnetic field.

The Hamiltonian of the system is:

$$H = H_o + H_{int}$$

$$H_o = \int d\tilde{x}\, \tilde{\phi}_s^+(\tilde{x}t)\left[\frac{\hbar^2}{2m}\left(-i\tilde{\nabla} + \frac{e}{c}\tilde{A}_{ext}\right)^2 + E_3\right]\tilde{\phi}_s(\tilde{x}t) \qquad (2.135)$$

Here the separation of the 3D wave operator (2.9) (2.10) has been carried out as before, and E_3 is the groundstate energy of the confining potential defined by (2.32). The interaction Hamiltonian H_{int} is of the same form as before (2.53). The Coulomb interaction of the electrons is neglected as will be justified later on.

The external magnetic field $\bar{B}$ is considered to lie in the x_3 direction, so the vector potential can be of the form:

$$\tilde{A} = (-By, 0), \quad A_3 = 0 \qquad (2.136)$$

here the notion $(x,y) \equiv (x_1,x_2)$ will be used for clarity. The eigen-functions of the free Hamiltonian H_o are now:

$$\phi_{nk}(\vec{x}) = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{\pi\ell^2}\right)^{1/4} \left(\frac{1}{n!2^n}\right)^{1/2} \exp\left[ikx - \frac{1}{2\ell^2}(y - k\ell^2)^2\right]$$

$$\cdot H_n\left(\frac{y - k\ell^2}{\ell}\right) \qquad (2.137)$$

with

$$\ell^2 = \frac{\hbar c}{eB}, \quad k = P_x/\hbar$$

where P_x is the eigen value of the momentum operator in the x-direction, n is the number of the Landau level. H_n stands for the n^{th} order Hermite polynomial. The eigenenergy of H_o is:

$$E_n = \hbar\omega_c(n + 1/2) \qquad (2.138)$$

where

$$\omega_c = \frac{eB}{mc} \qquad (2.139)$$

later on the groundstate $|\phi_o\rangle$ will be considered to have only one Landau level occupied, so the spin of the electrons can be neglected, and the field operators are defined as:

$$\hat{\phi}(\vec{x}t) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dk\ \hat{a}_{nk}(t)\ \phi_{nk}(\vec{x}) \qquad (2.140)$$

The creation and annihilation operators obey the usual anticommutation relation:

$$\{\hat{a}_{nk}(t), \hat{a}^{+}_{n'k'}(t')\}_{t=t'} = \delta_{n,n'}\ \delta(k - k') \qquad (2.141)$$

The number density of electrons n_e in any single fully occupied Landau level state $|\phi_n\rangle$ is (spinpolarized):

$$n_e = (\int d\tilde{x})^{-1} \langle \phi_n | \int d\tilde{x}\ \hat{\phi}^{+}\hat{\phi} | \phi_n \rangle = \frac{eB}{hc} \qquad (2.142)$$

The current of the free electrons is:

$$j_\mu^0(\tilde{x}t) = \frac{ie\hbar}{2m}\left[\hat{\phi}^{+}(\tilde{x}t)\ D_\mu\hat{\phi}(\tilde{x}t) - (D_\mu\hat{\phi}(\tilde{x}t))^{+}\hat{\phi}(\tilde{x}t)\right] \qquad (2.143)$$

where due to (2.136):

$$D_x = \partial_x - \frac{iy}{\ell^2}, \quad D_y = \partial_y \qquad (2.144)$$

Using the expansion (2.140) of the wave operator, the current becomes:

$$j_\mu(\tilde{x}t) = \frac{e\hbar}{2m\ell}\sum_{nn'}\int_{-\infty}^{\infty} dk dk' \Big[\phi^*_{n,k}(\tilde{x})\Big(\sigma(\mu)\sqrt{\frac{n'+1}{2}}\,\phi_{n'+1,k'}(\tilde{x}) + \sigma^*(\mu)\sqrt{\frac{n'}{2}}\,\phi_{n'-1,k'}(\tilde{x})\Big) + \Big(\sigma^*(\mu)\sqrt{\frac{n+1'}{2}}\,\phi^*_{n+1,k}(\tilde{x}) + \sigma(\mu)\sqrt{\frac{n}{2}}\,\phi^*_{n-1,k}(\tilde{x})\Big)\phi_{n',k'}(\tilde{x})\Big]\hat{a}^{+}_{nk}(t)\,\hat{a}_{n'k'}(t) \qquad (2.145)$$

where:

$$\sigma(x) = 1, \quad \sigma(y) = -i \qquad (2.146)$$

2.2.6 Evaluation of the Response Function, B≠0

Due to the loss of translational invariance of the system, caused by the external magnetic field, the response function:

$$iD^0_{\mu\nu}(x,x',t-t') = \theta(t-t')\langle\phi_0|\left[j^0_\mu(\tilde{x}t),\ j^0(\tilde{x}'t')\right]|\phi_0\rangle \qquad (2.147)$$

will not be Fourier transformed at the outset of its evaluation. For the same reason the projection operators (2.43) will not be used to express

the transverse response function as the response function of the transverse currents. The transverse and longitudinal response will have to be built up from the components of $D^{o}_{\mu\nu}$, and furthermore the transverse and longitudinal response functions will not be factorized as in the case of no external magnetic field (2.52). The transverse and longitudinal effects in a 2D system are therefore entangled by the external magnetic field.

Now eq. (2.145) is substituted into (2.147) and the following identity, for a groundstate $|\phi_o\rangle$ of only the lowest Landau level filled, is used.

$$\langle\phi|[\hat{a}^{+}_{nk}(t)\ \hat{a}_{n'k'}(t),\ \hat{a}^{+}_{mq}(t')\ \hat{a}_{m'q'}(t)]|\phi_o\rangle$$

$$= \delta_{n',m}\delta_{n,m'}\delta(k' - q)\ \delta(k - q')\Big[\, e^{-i\omega_c m(t - t')}\ \delta_{n,o}(1 - \delta_{m,o}) - e^{i\omega_c n(t - t')}\ \delta_{m,o}(1 - \delta_{n,o})\Big] \qquad (2.148)$$

Together with the integral,

$$\int_{-\infty}^{\infty} dk\ \phi_{nk}(\vec{x})\ \phi^{*}_{mk}(\vec{x}') = \frac{1}{2\pi\ell^2}\ \exp\left[\frac{i}{2\ell^2}(y + y')(x - x')\right] J_{nm}\left(\frac{|\vec{x} - \vec{x}'|}{\ell}\right) \qquad (2.149)$$

where:

$$J_{nm}\left(\frac{r}{\ell}\right) = \begin{cases} (-1)^{m-n}\left(\frac{n!}{m!}\right)^{1/2}\left(\frac{(y-y') + i(x-x')}{\sqrt{2}\ell}\right)^{m-n} L_n^{m-n}\left(\frac{r^2}{2\ell^2}\right)e^{-r^2/4\ell^2} \\ \qquad\qquad \text{if } m>n \\ \left(\frac{m!}{n!}\right)^{1/2}\left(\frac{(y-y') - i(x-x')}{\sqrt{2}\ell}\right)^{n-m} L_m^{n-m}\left(\frac{r^2}{2\ell^2}\right)e^{-r^2/4\ell^2} \\ \qquad\qquad \text{if } n>m \end{cases} \qquad (2.150)$$

with $r = |\vec{x}-\vec{x}'|$ and $L_n^m(x)$ is the Laguerre polynomial. Then $D^o_{\mu\nu}$ can be expressed as:

$$iD^o_{\substack{xx\\yy}}(\vec{r},\tau) = \frac{1}{2}\left(\frac{e\hbar}{2m\ell}\right)^2 \frac{e^{-r^2/2\ell^2}}{(2\pi\ell^2)^2} \sum_{n=1}^{\infty} \theta(\tau)\left[\pm \frac{r_y^2 - r_x^2}{\ell^2} \right.$$
$$\cdot \left(-L_n^2\left(\frac{r^2}{2\ell^2}\right) + 2L_{n-1}^2\left(\frac{r^2}{2\ell^2}\right) \right) + 4n\left(L_{n-1}\left(\frac{r^2}{2\ell^2}\right) - L_n\left(\frac{r^2}{2\ell^2}\right) \right.$$
$$\left. \left. + 2(n+1) L_{n+1}\left(\frac{r^2}{2\ell^2}\right) \right) \right] 2i\sin(-\omega_c n\tau) \qquad (2.151)$$

and:

$$iD^o_{\substack{xy\\yx}}(\vec{r},\tau) = \frac{1}{2}\left(\frac{e\hbar}{2m\ell}\right)^2 \frac{e^{-r^2}}{(2\pi\ell^2)^2} \sum_{n=1}^{\infty} \theta(\tau)\left[- \frac{2r_x r_y}{\ell^2} \right.$$
$$\cdot \left(-L_n^2\left(\frac{r^2}{2\ell^2}\right) + 2L_{n-1}^2\left(\frac{r^2}{2\ell^2}\right) \right) 2i\sin(-\omega_c n\tau)$$
$$\left. \mp 4n\left(L_{n-1}\left(\frac{r^2}{2\ell^2}\right) - L_n\left(\frac{r^2}{2\ell^2}\right) \right) 2i\cos(\omega_c n\tau) \right] \qquad (2.152)$$

where:

$$\vec{r} = \vec{x} - \vec{x}', \quad r_y = y - y', \quad r_x = x - x', \quad \tau = t - t' \qquad (2.153)$$

It is now evident that $D^o_{\mu\nu}$ is a function of $\vec{x} - \vec{x}'$ in spite of the broken translational invariance of the system. It is therefore possible to perform the Fourier transform with respect to $\vec{r}$ and τ. The Fourier transform with respect to $\vec{r}$ is defined as:

$$D^o_{..}(\vec{k},\tau) = \int_0^{\infty} r dr \int_0^{2\pi} d\phi\, e^{-ikr\cos(\phi - \phi_o)}\, D^o_{..}(\vec{r},\tau) \qquad (2.154)$$

where $\phi - \phi_o$ is the angle between $\vec{k}$ and $\vec{r}$. Therefore the following angular

integrals are needed:

$$\int_0^{2\pi} d\phi\, e^{-ikr\cos(\phi - \phi_0)} = 2\pi\, J_0(kr) \tag{2.155}$$

$$\int_0^{2\pi} d\phi \left(\sin^2\phi - \cos^2\phi \right) e^{-ikr\cos(\phi - \phi_0)} = 2\pi \frac{k_x^2 - k_y^2}{k^2}\, J_2(kr) \tag{2.156}$$

$$\int_0^{2\pi} d\phi\, \sin\phi\, \cos\phi\, e^{-ikr\cos(\phi - \phi_0)} = -2\pi \frac{k_x k_y}{k^2}\, J_2(kr) \tag{2.157}$$

where J_n is the Bessel function, and k_x and k_y are the x and y component of the wave vector $\tilde{k}$. The radial integration is of the form: (N.B.)

$$\int_0^{\infty} dx\, x^{\nu + 1}\, e^{-x^2}\, L_n^{\nu}(x^2)\, J_{\nu}(xy) = 2^{-\nu-1-2n}\, y^{\nu + 2n} \cdot e^{-y^2/4}\, \frac{1}{n!}\,. \tag{2.158}$$

The Fourier transform can be carried out by using (2.155-2.158) giving:

$$iD^0_{\substack{xx \\ yy}}(\tilde{k},\tau) = \left(\frac{e\hbar}{2m\ell}\right)^2 \frac{e^{-\frac{k^2\ell^2}{2}}}{(2\pi\ell^2)} \sum_{n=1}^{\infty} \theta(\tau) \left[\pm 2\, \frac{k_x^2 - k_y^2}{k^2}\, \frac{1}{(n-1)!} \left(\frac{\ell^2 k^2}{2}\right)^n \left(2 - \frac{1}{n}\left(\frac{\ell^2 k^2}{2}\right) \right) + \frac{4}{(n-1)!} \left(\frac{k^2\ell^2}{2}\right)^{n-1} \left(n - \left(\frac{k^2\ell^2}{2}\right) + \frac{1}{2n}\left(\frac{k^2\ell^2}{2}\right)^2 \right) \right] i\sin(-\omega_c n\tau) \tag{2.159}$$

In connection with this integral it should be mentioned that eq. (7.422.2) in (I. Gradsteyn 1965) is wrong and cannot be used to evaluate (2.158), instead one can use eq. (7.421.4) with the appropriate limit.

$$iD^{o}_{\substack{xy\\yx}}(\tilde{k},\tau) = \left(\frac{e\hbar}{2m\ell}\right)^2 \frac{e^{-\frac{k^2\ell^2}{2}}}{(2\pi\ell^2)} \sum_{n=1}^{\infty} \theta(\tau)\left[4\frac{k_x k_y}{k^2}\frac{1}{(n-1)!}\right.$$

$$\cdot \left(\frac{\ell^2k^2}{2}\right)^n \left(2 - \frac{1}{n}\left(\frac{\ell^2k^2}{2}\right)\right) i\sin(-\omega_c n\tau)$$

$$\left. \mp \frac{4}{(n-1)!}\left(\frac{k^2\ell^2}{2}\right)^{n-1}\left(n - \left(\frac{k^2\ell^2}{2}\right)\right) i\cos(\omega_c n\tau)\right] \qquad (2.160)$$

The transverse and longitudinal current response functions are defined as:

$$D^{ot}_{\mu\nu}(\tilde{k},\tau) \equiv \Pi_{\mu\lambda}(\tilde{k})\, D^{o}_{\lambda\nu}(\tilde{k},\tau)$$

$$D^{o\ell}_{\mu\nu}(\tilde{k},\tau) \equiv \mathbb{L}_{\mu\lambda}(\tilde{k})\, D^{o}_{\lambda\nu}(\tilde{k},\tau) \qquad (2.161)$$

From which it is seen that for example $D^{o\ell}_{xx}$ and D^{ot}_{xx} can be constructed as:

$$D^{o\ell}_{xx} = \frac{k_x k_x}{k^2} D^{o}_{xx} + \frac{k_x k_y}{k^2} D^{o}_{yx}$$

$$D^{ot}_{xx} = \left(1 - \frac{k_x k_x}{k^2}\right) D^{o}_{xx} - \frac{k_x k_y}{k^2} D^{o}_{yx} \qquad (2.162)$$

and similarly for the other components, leading to:

$$iD^{o\ell}_{\mu\nu}(\tilde{k},\tau) = i\mathbb{L}_{\mu\nu}(\tilde{k})\, H^{\ell}(k,\tau) + iO^{\ell}_{\mu\nu}(\tilde{k})\, F(k,\tau) \qquad (2.163)$$

and:

$$iD^{ot}_{\mu\nu}(\tilde{k},\tau) = i\Pi_{\mu\nu}(\tilde{k})\, H^{t}(k,\tau) + iO^{t}_{\mu\nu}(\tilde{k})\, F(k,\tau) \qquad (2.164)$$

where:

$$H^{\ell t}(k,\tau) = \left(\frac{e\hbar}{2m\ell}\right)^2 \frac{e^{-\frac{k^2\ell^2}{2}}}{2\pi\ell^2} \sum_{n=1}^{\infty} \left[\pm \frac{2}{(n-1)!} \left(\frac{\ell^2k^2}{2}\right)^n \right.$$

$$\cdot \left(2 - \frac{1}{n}\left(\frac{\ell^2k^2}{2}\right) \right) + \frac{4}{(n-1)!} \left(\frac{k^2\ell^2}{2}\right)^{n-1}$$

$$\left. \cdot \left(n - \left(\frac{k^2\ell^2}{2}\right) + \frac{1}{2n}\left(\frac{k^2\ell^2}{2}\right)^2 \right) \right] \sin(-\omega_c n\tau)\ \theta(\tau) \qquad (2.165)$$

and:

$$F(k,\tau) = \left(\frac{e\hbar}{2m\ell}\right)^2 \frac{e^{-\frac{k^2\ell^2}{2}}}{2\pi\ell^2} \sum_{n=1}^{\infty} \left[\frac{4}{(n-1)!} \left(\frac{k^2\ell^2}{2}\right)^{n-1} \right.$$

$$\left. \cdot \left(n - \left(\frac{k^2\ell^2}{2}\right) \right) \right] \cos(\omega_c n\tau)\ \theta(\tau) \qquad (2.166)$$

and the projection operators are:

$$O^{\ell}_{\substack{xx \\ yy}}(\tilde{k}) \equiv \pm\, \mathbb{I}_{xy}(\tilde{k}), \quad O^{\ell}_{\substack{xy \\ yx}}(\tilde{k}) \equiv \mp\, \mathbb{I}_{\substack{xx \\ yy}}(\tilde{k})$$

$$O^{t}_{\substack{xx \\ yy}}(\tilde{k}) \equiv \pm\, \pi_{xy}(\tilde{k}), \quad O^{t}_{\substack{xy \\ yx}}(\tilde{k}) \equiv \mp\, \pi_{\substack{xx \\ yy}}(\tilde{k}) \qquad (2.167)$$

The Fourier transform with respect to τ can be introduced now in the usual way:

$$D..(\tilde{k},\omega) = \int_{-\infty}^{\infty} d\tau\ e^{i\tau\omega}\ D..(\tilde{k},\tau) \qquad (2.168)$$

where the following integrals will be used:

$$\int_0^{\infty} d\tau\ e^{i\tau(\omega + i\eta)} \sin(-\omega_c n\tau) = \frac{\omega_c n}{\omega^2 - (\omega_c n)^2 + i\varepsilon} \qquad (2.169)$$

$$\int_0^\infty d\tau\, e^{i\tau(\omega + i\eta)} \cos(\omega_c n\tau) = \frac{i\omega}{\omega^2 - (\omega_c n)^2 + i\varepsilon} \qquad (2.170)$$

It is interesting to note here that due to the appearance of ω in the numerator of (2.170) and that only $F(k,\tau)$ has this $\cos(\omega_c n\tau)$ dependence, the following factorization is possible in the static limit ($\omega \to 0$):

$$D^{0\ell}_{\mu\nu}(\tilde{k},0) = \mathbb{L}_{\mu\nu}(\tilde{k})\, H^{\ell}(\tilde{k},0)$$

$$D^{0t}_{\mu\nu}(\tilde{k},0) = \mathbb{T}_{\mu\nu}(\tilde{k})\, H^{t}(\tilde{k},0) \qquad (2.171)$$

This factorization is possible for all values of $\tilde{k}$ and ω in the case of no external magnetic field.

The density response function can be derived by using (2.30) to relate it to the current response function (2.163) and (2.164). If the identities:

$$k_\mu k_\nu \mathbb{T}_{\mu\nu} = 0,\ k_\mu k_\nu \mathbb{L}_{\mu\nu} = k^2,\ k_\mu k_\nu (0^t_{\mu\nu} + 0^{\ell}_{\mu\nu}) = 0 \qquad (2.172)$$

are used then (2.30), (2.163) and (2.164) give:

$$H^{\ell}(k,\omega) = \frac{\omega^2}{k^2}\, D^0_{oo}(k,\omega) - \frac{e^2 n_e \hbar}{m} \qquad (2.173)$$

which can be rearranged to give:

$$D^0_{oo}(k,\omega) = \frac{e^{-\frac{k^2\ell^2}{2}}}{2\pi\ell^2}\, e^2 \sum_{n=1}^{\infty} \left(\frac{k^2\ell^2}{2}\right)^n \frac{1}{n!} \frac{2\omega_c n}{\omega^2 - (\omega_c n)^2 + i\eta} \qquad (2.174)$$

where $\eta \to 0^+$ and

$$n_e = \frac{1}{2\pi\ell^2} = \frac{eB}{hc} \qquad (2.175)$$

has been used. This result (2.174) is in an agreement with the results of C. Kallin and B.I. Halperin (C. Kallin, 1984).

From (2.163) and (2.164) it can be seen that a longwave limit of the current response ($k \to o$) is equivalent to restricting the Landau levels, in the infinite sum over n, to a finite number. It should also be noted that the choice of the groundstate $|\phi_o\rangle$ to have only the lowest Landau level occupied is equivalent to the high magnetic field approximation. The main difference between the response functions in an external field, and the ones in no magnetic field, besides of course the obvious quantization in to Landau levels when $B \neq 0$, is the fact that for the response functions in a magnetic field one can not factorize out the longitudinal and transverse projection operators, compare (2.163-4), (2.90) and (2.96).

2.2.7 2D Transverse Plasmon, $B \neq 0$

The question of collective modes in a q-2D electron system, in an external magnetic field, is a complex problem due to the possible mixing of transverse and longitudinal modes caused by the magnetic field. Therefore in this section the following specific problem will be studied; how does a purly transverse electromagnetic wave propagate in the system of q-2D electrons, interacting only via the vector potential?

K.W. Chiu and J.J. Quinn also neglected the Coulomb interaction when they (K.W. Chiu, 1974) derived the dispersion of the magneto plasmon in the high temperature semi classical approximation. The magneto plasmon represents a mixture of longitudinal and transverse modes.

As before the interaction Hamiltonian will be considered to be:

$$H_{int} = -\frac{1}{c}\int d\bar{x}\; A_\nu(\bar{x}t)\; \hat{J}_\nu(\bar{x}t) - \frac{e^2}{2mc}\int d\bar{x}\; A_\nu^2\; \hat{\psi}^+\hat{\psi} \qquad (2.176)$$

the second term was neglected by K.W. Chiu and J.J. Quinn (K.W. Chiu, 1974). The dynamics of A_ν, in the Coulomb gauge, are described as before by the wave eq.

$$\left[\bar{\nabla}^2 - \frac{1}{c^2}\partial_t^2\right] A_\nu(\bar{x},t) = -\frac{4\pi}{c}\langle \hat{j}^t_\nu(\bar{x},t)\rangle \qquad (2.177)$$

The vector potential A_ν in (2.176) and (2.177) is the internal self consistent field, while the effects of the external constant magnetic field are accounted for in the free Hamiltonian H_o and the wavefunction together with the groundstate.

Just as in section (2.2.2) the self consistent linear response of the 2D electrons is:

$$\langle j^t_\mu(\tilde{k},\omega)\rangle = \frac{e^2 n_e}{2\pi mc}\,\Gamma(k,\omega)\; j^t_\mu(k,\omega) - \frac{1}{2\pi\hbar c}\,\Gamma(k,\omega) \cdot D^{ot}_{\mu\nu}(\tilde{k},\omega)\langle j^t_\nu(\tilde{k},\omega)\rangle \qquad (2.178)$$

where $\Gamma(k,\omega)$ is given by (2.78). The external vector potential (2.136) poses no difficulty to the dimensional reduction since it is entirely two dimensional.

The second term on the r.h.s. in (2.178) has the form:

$$-\frac{1}{2\pi\hbar c}\,\Gamma(k,\omega)\; D^{ot}_{\mu\nu}(\tilde{k},\omega)\langle j^t_\nu(\tilde{k},\omega)\rangle = \frac{1}{2\pi\hbar c}\,\Gamma(k,\omega)\left[\Pi_{\mu\nu}(\tilde{k})\; H^t(k,\omega) + 0^t_{\mu\nu} F(k,\omega)\right] \cdot \Pi_{\nu\lambda}(\tilde{k})\langle J_\lambda(\tilde{k},\omega)\rangle \qquad (2.179)$$

where eq. (2.164) has been used for $D^{ot}_{\mu\nu}$. From the definition of $0^t_{\mu\nu}$ (2.167) it becomes clear that:

$$0^{t}_{\mu\nu}\ \Pi_{\nu\lambda} = 0 \qquad (2.180)$$

thus leading to:

$$\left[1 + \left(\frac{e^2 n_e}{2\pi mc} + \frac{1}{2\pi\hbar c}\ H^{t}(k,\omega)\right)\Gamma(k,\omega)\right] <j^{t}_{\nu}(\vec{k},\omega)> = 0 \qquad (2.181)$$

So inspite that in the presence of an external magnetic field the transverse projection operator cannot be factorized out of $D^{ot}_{\mu\nu}$, the identity (2.180) leads to the expression (2.181). Which is of the same functional form as the corresponding in one in the case of no external magnetic field (2.76). The dispersion relation of a transverse electromagnetic wave obtained from (2.181) is:

$$\frac{1}{\Gamma(k,\omega)} + \frac{e^2 n_e}{2\pi mc} + \frac{1}{2\pi\hbar c}\ H^{t}(k,\omega) = 0 \qquad (2.182)$$

where eq. (2.165) and (2.175) give:

$$H^{t}(k,\omega) = \frac{e^2 \hbar n_e}{4m}\, e^{-\frac{k^2\ell^2}{2}} \sum_{n=1}^{\infty} \frac{2}{(n-1)!}\left[2n\left(\frac{\ell^2 k^2}{2}\right)^{n-1} - 4\left(\frac{\ell^2 k^2}{2}\right)^{n} + \frac{2}{n}\left(\frac{\ell^2 k^2}{2}\right)^{n+1}\right]$$

$$\cdot \frac{\omega_c^2 n}{\omega^2 - (\omega_c n)^2 + i\eta}, \qquad \eta \to 0^{+} \qquad (2.183)$$

If the magnetic field B is strong enough, such that only one Landau level above the filled ground level participates in the collective mode then:

$$H^t(k,\omega) = \frac{e^2\hbar n_e}{m} e^{-\frac{k^2\ell^2}{2}} \left[1 - 2\left(\frac{\ell^2k^2}{2}\right) + \left(\frac{\ell^2k^2}{2}\right)^2 \right]$$

$$\frac{\omega_c^2}{\omega^2 - \omega_c^2 + i\eta}$$

$$= \frac{e^2\hbar n_e}{m} \left(1 - 3\left(\frac{\ell^2k^2}{2}\right) \right) \frac{\omega_c^2}{\omega^2 - \omega_c^2 + i\eta} + O\left(\left(\frac{\ell^2k^2}{2}\right)^2\right) \qquad (2.184)$$

Due to the structure of eq. (2. 183) this high magnetic field limit is equivalent to the long wavelength limit k→0. Substitution of (2.184) into (2.182) gives the dispersion relation:

$$\frac{1}{\Gamma(k,\omega)} + \frac{e^2n_e}{2\pi mc} + \frac{e^2n_e}{2\pi mc} \left(1 - 3\left(\frac{\ell^2k^2}{2}\right) \right) \frac{\omega_c^2}{\omega^2 - \omega_c^2 + i\eta} = 0 \qquad (2.185)$$

And if the wave function $\rho(k_3)$ is assumed to be a Gaussian (2.123):

$$\rho(k_3) = e^{-k_2^2/2a^2} \qquad (2.186)$$

then $\Gamma(k,\omega)$ (2.125) becomes:

$$\Gamma(k,\omega) = -\frac{4\pi\sqrt{\pi}\, ac}{\omega^2 - c^2k^2} + O\left(\left(\frac{ac}{\omega}\right)^3\right) \qquad (2.187)$$

Thus the dispersion (2.185) is:

$$\left[\frac{\omega^2 - c^2k^2}{4\pi\sqrt{\pi}\, ac} - \frac{e^2n_e}{2\pi mc} \right] (\omega^2 - \omega_c^2) = \frac{e^2n_e}{2\pi mc} \omega_c^2 \left(1 - 3\left(\frac{\ell^2k^2}{2}\right) \right) \qquad (2.188)$$

which is equivalent to:

$$\omega^4 - \omega^2\left[c^2k^2 + \frac{e^2n_e}{m}\,2\sqrt{\pi}\,a\right] + \omega_c^2\left[c^2k^2 + 3\,\frac{e^2n_e}{m}\,(\frac{\ell^2k^2}{2})\,2\sqrt{\pi}\,a\right] = 0 \tag{2.189}$$

This second order polynomial in ω^2 has the positive root:

$$\omega^2 = \frac{1}{2}\left[c^2k^2 + \frac{e^2n_e}{m}\,2\sqrt{\pi}\,a\right] + \frac{1}{2}\left[\left[c^2k^2 + \frac{e^2n_e}{m}\,2\sqrt{\pi}\,a\right]^2 - 4\omega_c^2\left[c^2k^2 + 3\,\frac{e^2n_e}{m}\,(\frac{\ell^2k^2}{2})\,2\sqrt{\pi}\,a\right]\right]^{1/2} \tag{2.190}$$

in deriving the dispersion up till now, the following limits have been used:

$$(\frac{\ell^2k^2}{2}) \ll 1, \quad (\frac{ac}{\omega}) \ll 1 \tag{2.191}$$

It is remarkable to notice that, even though (2.190) is derived in the strong B limit (2.191), that (2.190) gives:

$$\omega^2 = c^2k^2 + \frac{2\pi e^2n_e}{m}\,(\frac{a}{\sqrt{\pi}}) \tag{2.192}$$

in the not proper limit B→0. This result is the same as that obtained for the case of no external magnetic field (2.126) for the transverse plasmon (up to $O(k^2)$).

The dispersion relation (2.190) can be written as:

$$\omega^2 = \frac{1}{2}(c^2k^2 + \omega_t^2) + \frac{1}{2}(c^2k^2 + \omega_t^2) \cdot \left[1 - \frac{4\omega_c^2\left[c^2k^2 + 3(\frac{\ell^2k^2}{2})\omega_t^2\right]}{(c^2k^2 + \omega_t^2)^2}\right]^{1/2} \tag{2.193}$$

where:

$$\omega_t^2 \equiv \frac{2\pi e^2 n_e}{m} \left(\frac{a}{\sqrt{\pi}}\right) \tag{2.194}$$

For k sufficiently small k→0 (2.193) becomes:

$$\omega^2 = \omega_t^2 + c^2k^2 - \frac{\omega_c^2\left[c^2k^2 + 3\left(\frac{\ell^2k^2}{2}\right)\omega_t^2\right]}{c^2k^2 + \omega_t^2} \tag{2.195}$$

with:

$$\left(\frac{\ell^2k^2}{2}\right)<<1, \quad \left(\frac{ac}{\omega}\right)<<1, \quad k\to 0 \tag{2.196}$$

where the dispersion has the same gap ω_t as in the case of no external magnetic field (2.126)

$$\omega(k = 0) = \omega_t \tag{2.197}$$

SUMMARY

In this section a general formalism has been presented to derive the effective Hamiltonian for the q-2D electron gas from the full three dimensional Hamiltonian of the system. The important finite thickness of the q-2D electron gas has thus been taken into account, in contrast to most existing works on 2D electron gas.

For the first time the current response functions for the system, with or without an external magnetic field, have been separated into the longitudinal and the transverse parts by general projection operators. The linear response of the transverse current to the vector potential $\bar{A}$ has then been used consistently, with the wave-equation for $\bar{A}$, to derive the dispersion relation for the propagation of the transverse electro-

magnetic field in the system, the transverse plasmon. The analytic results for the dispersion, derived under very restrictive conditions for the parameters of the system, where shown to depend crucially on the thickness of the 2D electron gas. These 2D transverse plasmons have not been studied before.

One can see several interesting directions for future research. The transverse response functions may be of importance in q-2D superconductors (H. Takagi, 1982a, b, 1983), since in the Meissner effect an external transverse vector field $\bar{A}$ induces a transverse current in the superconductor.

Due to the rather restrictive conditions, (2.196) and (2.129), under which the analytical dispersion relations (2.130), (2.131) and (2.195) are derived, it is of great importance to solve the general dispersion relations (2.77) and (2.182) numerically. In this connection, the effect of including more than one energy level of the confining potential, needs to be studied. The plasmon dispersion can also be studied at finite temperature, especially in order to derive the damping effects.

It has been noticed (T. Toyoda, 1984d) that when the magnetic field is strong enough so only one Landau level is partially occupied the current is almost entirely transverse. It is therefore an open question whether the transverse mode in a magnetic field has any connection with the FQHE, which is observed under these same conditions.

2.3 MICROSCOPIC THEORY OF THE IQHE

Although most theoretical works on the QHE are now concerned with the FQHE, the microscopic mechanism of the IQHE has not been fully understood. This section will begin with a simple classical model of the Hall

effect, in order to define the relevant observables, and clear some of the confusion about what is measured in experiments. It is then shown that the Hall conductance σ_H can be derived from the quantum model (2.3) by QFTMB methods, giving results which are analogous to the classical results in form. The Hall conductance σ_H, expressed by one formula, will be able to account for the temperature dependence, the spin and valley splitting, the finite width of Landau levels and the plateau width of the QHE. On the other hand it is independent of the exact mechanism of electron localization, which is controversial at the time being.

CHE

The electrons in a 2D system (the (x-y)-plane) are subjected to an external constant magnetic field $\bar{B} = (0,0,B)$, and an external electric field $\bar{E} = (E,0,0)$, (see Fig. 2). In a finite system the Lorentz force on the electrons, which are travelling in the x direction, can deflect them in the negative y direction, thereby creating an internal electric field E_y. The classical equation of motion for one electron is (N.W. Ashcroft, 1976)

$$d_t\bar{p} = -e(\bar{E} + \frac{\bar{p}}{mc} \times \bar{B}) - \frac{\bar{p}}{\tau} \qquad (2.198)$$

where $\bar{p}$ is the momentum of the electron, and the last term is a phenomenological damping term, where τ is the relaxation time between collisions.

Due to the damping term it is possible to assume that the system is in a steady state, $\dot{\bar{p}} = 0$. Then eq. (2.198) gives:

$$\begin{aligned} 0 &= -eE_x - \omega_c p_y - \frac{p_x}{\tau} \\ 0 &= -eE_y + \omega_c p_x - \frac{p_y}{\tau} \end{aligned} \qquad (2.199)$$

where ω_c is the cyclotron frequency:

$$\omega_c = \frac{eB}{mc} \tag{2.200}$$

If the 2D current density $\tilde{j}$ is defined:

$$\tilde{j} = -n_e e \tilde{v} \tag{2.201}$$

where $\tilde{v}$ is the velocity and n_e is the 2D density of electrons, together with the classical conductivity σ_o

$$\sigma_o = \frac{n_e e^2 \tau}{m} \tag{2.202}$$

then eq.'s (2.199) give:

$$\begin{aligned} \sigma_o E_x &= \omega_c \tau\, j_y + j_x \\ \sigma_o E_y &= -\omega_c \tau\, j_x + j_y \end{aligned} \tag{2.203}$$

These equations can be conveniently written in a tensor form:

$$\begin{aligned} E_\mu &= \rho_{\mu\nu}\, j_\nu \\ j_\mu &= \sigma_{\mu\nu}\, E_\nu \end{aligned} \tag{2.204}$$

where the resistivity tensor $\rho_{\mu\nu}$ is:

$$\tilde{\rho} = \frac{1}{\sigma_o} \begin{bmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{bmatrix} \tag{2.205}$$

and the conductivity tensor $\sigma_{\mu\nu}$ (the inverse of $\rho_{\mu\nu}$) has the form:

$$\tilde{\sigma} = \frac{\sigma_o}{1 + (\omega_c \tau)^2} \begin{bmatrix} 1 & -\omega_c \tau \\ \omega_c \tau & 1 \end{bmatrix} \tag{2.206}$$

From (2.205) and (2.206) one finds:

$$\sigma_{xy} = - \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}, \quad \sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \tag{2.207}$$

which implies that if ρ_{xx} does vanish then σ_{xx} also vanishes at the same time. Relations can also be found between σ_{xx} and σ_{xy}, which will be of importance later.

$$\sigma_{xy} = - \frac{\sigma_o \omega_c \tau}{1 + (\omega_c \tau)^2} = - \omega_c \tau \, \sigma_{xx} \tag{2.208}$$

and:

$$\sigma_{xy} = - \frac{\sigma_o \omega_c \tau}{1 + (\omega_c \tau)^2} = - \frac{\sigma_o (1 + \omega_c^2 \tau^2) + \sigma_o}{\omega_c \tau (1 + \omega_c^2 \tau^2)}$$

$$= \frac{\sigma_o}{\omega_c \tau} + \frac{\sigma_o}{\omega_c \tau} \frac{1}{1 + \omega_c^2 \tau^2}$$

$$= \frac{n_e e c}{B} + \frac{1}{\omega_c \tau} \sigma_{xx} \tag{2.209}$$

where eq.'s (2.200) and (2.202) have been used.

The experimental 2D electron systems are finite, so initially the Lorentz force on the electrons, travelling in the x-direction will create a current j_y. No current can flow out of the system in the y-direction, so after a short time the charge imbalance will create an electrical field E_y which will counteract the Lorentz force. When the system has attained equilibrium then:

$$j_y = 0 \tag{2.210}$$

This important condition (2.210) leads to the Hall effect. Equation

(2.210) gives a relation between E_x and E_y:

$$j_y = \sigma_{yy}E_y + \sigma_{yx}E_x = 0 \qquad (2.211)$$

therefore:

$$\frac{E_x}{E_y} = -\frac{\sigma_{yy}}{\sigma_{yx}} \qquad (2.212)$$

The Hall conductivity σ_H can now be calculated as:

$$\sigma_H \equiv \frac{j_x}{E_y} = \sigma_{xx}\frac{E_x}{E_y} + \sigma_{xy} = \frac{\sigma_{xx}^2}{\sigma_{xy}} + \sigma_{xy} \qquad (2.213)$$

where (2.212) was used. If now eq. (2.208) and (2.209) are used in (2.213) then:

$$\sigma_H = -\frac{\sigma_{xx}}{\omega_c\tau} + \sigma_{xy} = -\frac{n_e ec}{B} \qquad (2.214)$$

The Hall conductivity σ_H is also related to ρ_{yx} by eq.(2.204) and (2.210):

$$\sigma_H = (\rho_{yx})^{-1} = -\frac{n_e ec}{B} \qquad (2.215)$$

By similar methods one finds:

$$(\rho_{xx})^{-1} = \frac{j_x}{E_x} = \sigma_o \qquad (2.216)$$

It is therefore clear that, in an experiment where $j_y = 0$, ρ_{xx} and ρ_{yx} are measured and eq.(2.215) holds independent of the value of ρ_{xx}. The discussion of T. Ando and H. Aoki (H. Aoki, 1981) (T. Ando, 1975) about the plateaus in σ_{xy}:

$$\sigma_{xy} = -\frac{n_c ec}{B} + \Delta\sigma_{xy} \qquad (2.217)$$

developing only when $\sigma_{xx} = 0$, and therefore $\Delta\sigma_{xy} = 0$, is thus irrelevant

to experiments where $j_y = 0$.

In the case when $E_y = 0$ in experiments instead of the usual condition (2.210) then:

$$\frac{j_y}{E_x} = \sigma_{yx} = \frac{n_e ec}{B} - \frac{1}{\omega_c \tau}\sigma_{xx}$$

$$\frac{j_x}{E_x} = \sigma_{xx} \tag{2.218}$$

so here the theory of T. Ando and H. Aoki is relevant. The only experiments carried out under this condition are by j. Wakabayashi and S. Kawaji et. al. (j. Wakabayashi, 1980a, b)(S. Kawaji, 1975, 1976).

IQHE

It will now be shown that the formula (2.215) for the Hall conductivity σ_H can be derived from a QFTMB-model, where of course n_e will be replaced by its grand canonical ensemble average.

The full 3D Hamiltonian of the q-2D electrons is:

$$H = H_o' + H_{int} \tag{2.219}$$

where:

$$H_o' = H_o + H_{spin} + H_{Coul} + H_{confining} \tag{2.220}$$

and:

$$H_{int} = H_E + H_{imp} \tag{2.221}$$

H_o is given by:

$$H_o = \int d\bar{x}\, \hat{\psi}_s^{+} \left[-\frac{1}{2m}\left(\bar{\nabla} + \frac{ie}{\hbar c}\bar{A}(\bar{x})\right)^2 - \mu \right] \hat{\psi}_s(\bar{x}t) \tag{2.222}$$

where μ is the chemical potential and $\bar{A}(\tilde{x})$ is the vector field

$\bar{A}(\bar{x}) = (-B_y, o, o)$ causing the external constant magnetic field $\bar{B} = (o,o,B)$. The spin Hamiltonian is as before (2.8):

$$H_{spin} = \frac{1}{2} g\mu_B \sum_{i=1}^{3} \int dx\ \hat{\psi}_s^+(\bar{x}t) \left| \bar{\nabla} \times A(\bar{x}) \right|_i (\sigma_i)_{sr} \hat{\psi}(\bar{x}t) \qquad (2.223)$$

H_{coul} is given by (2.6), while $H_{confining}$ is:

$$H_{confining} = \int d\bar{x}\ \hat{\psi}_s^+(\bar{x}t)\ V(x_3)\ \hat{\psi}_s(\bar{x}t) \qquad (2.224)$$

where $V(x_3)$ is the confining potential of the q-2D electron system. The impurity Hamiltonian is given by (2.7) and H_E is:

$$H_E = - e\bar{E} \cdot \int d\bar{x}\ \hat{\psi}_s^+(\bar{x}t)\bar{x}\ \hat{\psi}_s(\bar{x}t) \qquad (2.225)$$

where E_x is the external electric field causing the conduction in the x-direction. E_y is the induced electric field due to the Lorentz-force and the finite sample size.

As before (2.9) the dimensional reduction down to an effective 2D Hamiltonian will be accomplished by the factorization:

$$\hat{\psi}_s^+(\bar{x}t) = \sum_n \hat{\phi}_s^+(\bar{x}t)\ \chi_n(x_3) \qquad (2.226)$$

where χ_n is the wavefunction of the nth level of the confining potential $V(x_3)$:

$$\left[- \frac{\hbar^2}{2m} \partial_3^2 + V(x_3) \right] \chi_n(x_3) = (E_3)_n\ \chi_n(x_3) \qquad (2.227)$$

and $(E_3)_n$ is the eigenenergy of χ_n. In experiments on the IQHE the electrons are usually assumed to be confined to the lowest level $\chi_o = \chi$.

The effective 2D Hamiltonians now become:

$$H_o = \int d\tilde{x}\ \hat{\phi}_s^+(\tilde{x}t)\left[-\frac{\hbar^2}{2m}\left(\tilde{\nabla} + \frac{ie}{\hbar c}\tilde{A}(\tilde{x}t)\right)^2 - \mu + E_3\right]\hat{\phi}_s(\tilde{x}t)$$

$$= \int d\tilde{x}\ \hat{\phi}_s^+(\tilde{x}t)\left[-\frac{\hbar^2}{2m}(D_x^2 + D_y^2) - \mu + E_3\right]\hat{\phi}_s(\tilde{x}t) \qquad (2.228)$$

where:

$$D_x = \partial_x - \frac{iy}{\ell^2}, \quad D_y = \partial_y, \quad \ell^2 = \frac{eB}{\hbar c} \qquad (2.229)$$

This effective 2D H_o is a combination of the 3D H_o and $H_{confining}$. Since the external A-field is two dimensional the spin Hamiltonian is simply:

$$H_{spin} = \frac{1}{2}g\mu_B \sum_{i=1}^{3}\int d\tilde{x}\ \hat{\phi}_s^+(\tilde{x}t)\ B(\sigma_z)_{sr}\ \hat{\phi}^+(\tilde{x}t) \qquad (2.230)$$

and:

$$H_E = -e\ E_\nu \int d\tilde{x}\ \hat{\phi}_s^+(\tilde{x}t)\ x_\nu\ \hat{\phi}_s(\tilde{x}t), \quad (\nu = 1,2) \qquad (2.231)$$

The main impact of the dimensional reduction is seen in the Coulomb and the impurity interaction Hamiltonians, H_{coul} and H_{imp}.

The effective 2D H_{imp} is:

$$H_{imp} = \sum_i \int d\tilde{x}\ \hat{\phi}_s^+(\tilde{x}t)\ v(\tilde{x} - \tilde{x}_i, z_i)\ \hat{\phi}_s(\tilde{x}t) \qquad (2.232)$$

where:

$$v(x - x_i, z_i) = \int_{-\infty}^{\infty} dx_3\ \chi(x_3)\ V(\bar{x} - \bar{x}_i)\ \chi(x_3) \qquad (2.233)$$

is the effective impurity potential of the scatterer located at $\bar{x}_i = (\tilde{x}_i, z_i)$, only one level of the confining potential is considered occupied, and one type of impurity is assumed present.

The effective 2D H_{coul} is:

$$H_{coul} = \frac{1}{2}\int d\tilde{x}d\tilde{x}'\ \hat{\phi}_s^+(\tilde{x}t)\ \hat{\phi}_r^+(\tilde{x}'t)\ U(\tilde{x} - \tilde{x}')\ \hat{\phi}_r(\tilde{x}'t)\ \hat{\phi}_s(\tilde{x}t) \quad (2.234)$$

where the effective 2D Coulomb interaction has the form:

$$U(\tilde{x} - \tilde{x}') = \int_{-\infty}^{\infty} dx_3 dx_3'\ \chi(x_3)\ \chi(x_3)\ \chi(x_3')\ \chi(x_3')\ U(\bar{x} - \bar{x}') \quad (2.235)$$

where:

$$U(\bar{x} - \bar{x}') = \frac{e^2}{|\bar{x} - \bar{x}'|} \quad (2.236)$$

Generally in works on q-2D systems attention has not been given to the effective Coulomb interaction which can have a different form from the usually chosen Coulomb interaction of a 2D electron gas

$$U(\tilde{x} - \tilde{x}') = \frac{e^2}{|\tilde{x} - \tilde{x}'|} \quad (2.237)$$

Analogously to (2.219) the effective 2D Hamiltonian is divided into the "free" and interacting parts:

$$H_o = H_o' + H_{int} \quad (2.238)$$

where:

$$H_o' = H_o + H_{spin} + H_{coul}$$
$$H_{int} = H_E + H_{imp} \quad (2.239)$$

The current density for the 3D Hamiltonian is given by:

$$\hat{J}_k(\bar{x}t) = \frac{ie\hbar}{2m}\left[\hat{\psi}_s^+(\bar{x}t)\,\partial_k\,\hat{\psi}_s(\bar{x}t) - (\partial_k\hat{\psi}_s^+(\bar{x}t))\,\hat{\psi}_s(\bar{x}t)\right]$$

$$- \frac{e^2}{mc}\,\bar{A}_k(\bar{x})\hat{\psi}_s^+(\bar{x}t)\,\hat{\psi}_s(\bar{x}t)$$

$$- \frac{g\mu_B c}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\epsilon_{kji}\,\partial_j\left[\hat{\psi}_s^+(\bar{x}t)(\sigma_i)_{sr}\,\hat{\psi}_r(\bar{x}t)\right] \qquad (2.240)$$

See (2.11b), ϵ_{kji} is the antisymmetric Levi-Civita tensor and σ_i is the i^{th} Pauli spin matrix. The current is defined as:

$$\hat{I}_k^{3D} = \int d\bar{x}\,\hat{J}_k(\bar{x}t), \quad (k = 1,2,3) \qquad (2.241)$$

If the wavefunction $\chi(x_3)$ (or χ_n) is real and since the spin current is expressed as a divergence of a quantity then:

$$\hat{I}_3^{3D} = 0 \qquad (2.242)$$

In the other two components $\hat{I}_1^{3D}$ and $\hat{I}_2^{3D}$ the x_3 integration can be performed since $\bar{A}(\bar{x}) = (-By,o,o)$. Therefore the 2D current is:

$$\hat{I}_\mu(t) = \int d\tilde{x}\,\hat{j}_\mu(\tilde{x}t), \quad (\mu = 1,2) \qquad (2.243)$$

$$= \frac{ie\hbar}{2m}\int d\tilde{x}\left[\hat{\phi}_s^+(\tilde{x}t)D_\mu\,\hat{\phi}_s(\tilde{x}t) - (D_\mu\hat{\phi}_s(\tilde{x}t))^+\,\hat{\phi}_s(\tilde{x}t)\right] \qquad (2.244)$$

where D_μ is defined by (2.229). The 2D density operator is defined as:

$$\hat{\rho}(\tilde{x}t) = -e\,\hat{\phi}_s^+(\tilde{x}t)\,\hat{\phi}_s(\tilde{x}t) \qquad (2.245)$$

In order to obtain an equation of a similar form as the classical equation of motion for the momentum $\bar{p}$ (2.198), one can use the following canonical equation of motion for the current (2.243):

$$d_t \hat{I}_\mu(t) = \frac{1}{i\hbar}[\hat{I}_\mu(t), H] \tag{2.246}$$

This commutator of the total effective 2D Hamiltonian and the current can be evaluated using the canonical commutation relation (2.10) and the formulae:

$$\begin{aligned}[\hat{\phi}_s^+(\tilde{x}t)(\partial_\mu - \overleftarrow{\partial}_\mu)\,\hat{\phi}_s(\tilde{x}t),\ \hat{\phi}_r^+(\tilde{x}'t)\,F(\partial')\,\hat{\phi}_r(\tilde{x}'t)] \\ = \delta_{s,r}\left\{\hat{\phi}_s^+(\tilde{x}t)(\partial_\mu - \overleftarrow{\partial}_\mu)\,\delta(\tilde{x} - \tilde{x}')\right\} F(\partial')\,\hat{\phi}_s(\tilde{x}'t) \\ - \delta_{s,r}\,\hat{\phi}_s^+(\tilde{x}'t)\left\{(F(\partial')\,\delta(x - x'))\,(\partial_\mu - \overleftarrow{\partial}_\mu)\,\hat{\phi}_s(\tilde{x}t)\right\}\end{aligned} \tag{2.247}$$

and:

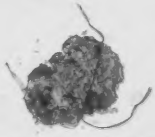
$$\begin{aligned}[\hat{\phi}_s^+(\tilde{x}t)\,\hat{\phi}_s(\tilde{x}t),\ \hat{\phi}_r^+(\tilde{x}'t)\,F(\partial')\,\hat{\phi}_r(\tilde{x}'t)] \\ = \delta_{s,r}\,\delta(\tilde{x} - \tilde{x}')\,\hat{\phi}_s^+(\tilde{x}t)\,F(\partial')\,\hat{\phi}_s(\tilde{x}'t) \\ - \delta_{s,r}\,\{F(\partial')\,\delta(\tilde{x} - \tilde{x}')\}\,\hat{\phi}_s^+(\tilde{x}'t)\,\hat{\phi}_s(\tilde{x}t)\end{aligned} \tag{2.248}$$

where $F(\partial')$ is any differential operator.

The commutator (2.246) then gives:

$$\begin{aligned}d_t \hat{I}_1(t) = &- \omega_c \hat{I}_2 + \frac{e^2}{m} E_1 \int d\tilde{x}\,\rho(\tilde{x}t) \\ &+ \frac{e}{m}\int d\tilde{x}d\tilde{x}'(\partial_1 U(\tilde{x} - \tilde{x}'))\,\hat{\rho}(\tilde{x}t)\,\hat{\rho}(\tilde{x}'t) \\ &+ \frac{1}{i\hbar}[\hat{I}_1(t), H_{imp}]\end{aligned} \tag{2.249}$$

and:



$$d_t \hat{I}_2(t) = \omega_c \hat{I}_1 + \frac{e^2}{m} E_2 \int d\tilde{x}\, \hat{\rho}(\tilde{x}t)$$

$$+ \frac{e}{m} \int d\tilde{x} d\tilde{x} (\partial_2 U(\tilde{x} - \tilde{x}')) \hat{\rho}(\tilde{x}t)\, \hat{\rho}(\tilde{x}'t)$$

$$+ \frac{i}{\hbar} [\hat{I}_2(t), H_{imp}] \qquad (2.250)$$

The above equations are for the Heisenberg operators. Now a proper quantum statistical mechanical average for these operators as well as a proper random distribution for the impurities should be introduced. The latter is necessary to obtain the dissipation term for the current. The dissipation term will not be derived here, instead the results from the literature will be adopted (A.A. Abrikosov, 1963) (S. Doniach, 1974). That is, the last terms in (2.249) and (2.250) cause the dissipation after the quantum statistical mechanical ensemble average and the impurity distribution average are taken. The dissipation is characterized by the relaxation time $\tau = \tau(B, n_e, T, \ldots\ldots)$, which will not appear explicitly in the formula for the Hall conductivity. This two stepped average procedure will be denoted by $\langle \ldots\ldots \rangle$. Then, the equation of motion for the current expectation value becomes:

$$d_t \langle \hat{I}_1(t) \rangle = -\omega_c \langle \hat{I}_2(t) \rangle + \frac{e^2}{m} E_1 \int d\tilde{x} \langle \hat{\rho}(\tilde{x}t) \rangle$$

$$+ \frac{e}{m} \int d\tilde{x} d\tilde{x}' (\partial_1 U(\tilde{x} - \tilde{x}')) \langle \hat{\rho}(\tilde{x}t)\, \hat{\rho}(\tilde{x}'t) \rangle - \frac{1}{\tau} F[\langle \hat{I}_1 \rangle] \qquad (2.251)$$

and:

$$d_t \langle \hat{I}_2(t) \rangle = \omega_c \langle \hat{I}_1(t) \rangle + \frac{e^2}{m} \int d\tilde{x} \langle \rho(\tilde{x}t) \rangle$$

$$\frac{e}{m} \int d\tilde{x} d\tilde{x}' (\partial_2 U(\tilde{x} - \tilde{x}')) \langle \hat{\rho}(\tilde{x}t)\, \hat{\rho}(\tilde{x}'t) \rangle - \frac{1}{\tau} F[\langle \hat{I}_2 \rangle] \qquad (2.252)$$

where $F[<\hat{I}_2>] = F[<\hat{I}_2>, <\hat{I}_1>.....]$ is any function of $<\hat{I}_2>$ describing the dissipation, and $F[<\hat{I}_2> = 0] = 0$. F could for example be the identity function as in the classical derivation. Considering the symmetry of the last part of (2.251) and (2.252) one can easily see that it vanishes. Therefore the Coulomb interaction has no effect on the equations of motion. Thus the equations of motion are:

$$d_t<\hat{I}_1(t)> = -\omega_c<\hat{I}_2(t)> + \frac{e^2}{m} E_1 \int d\tilde{x}<\hat{\rho}(\tilde{x}t)> - \frac{1}{\tau} F[<\hat{I}_1>] \qquad (2.254)$$

$$d_t<\hat{I}_2(t)> = \omega_c<\hat{I}_1(t)> + \frac{e^2}{m} E_2 \int d\tilde{x}<\hat{\rho}(\tilde{x}t)> - \frac{1}{\tau} F[<\hat{I}_2>] \; . \qquad (2.255)$$

In order to obtain the formula for the Hall conductivity, the following conditions must be imposed:

$$d_t<\hat{I}_1(t)> = d_t<\hat{I}_2(t)> = 0 \qquad (2.256)$$

$$<\hat{I}_2> = 0 \; , \longrightarrow F[<I_2>] = 0 \; . \qquad (2.257)$$

The condition (2.257) is very important to interpret the experiment by K.v. Klitzing et al. (K.v. Klitzing, 1980, 1981). With these conditions (2.256) and (2.257), the equations of motion, (2.254) and (2.255), for the current expectation value can be compared to the classical equations (2.203). So as in the classical case, if the current density and the number density expectation values are uniform in space, then:

$$\sigma_H = \frac{<\hat{I}_1>}{\int d\tilde{x}} \frac{1}{E_2} = - \frac{n_e ec}{B} \qquad (2.258)$$

where $n_e \equiv \langle \hat{\rho} \rangle$. This expression is independent of the value for ρ_{xx}. The averaging procedure for calculating the expectation value of the number density operator in (2.258) has not been specified. In the MOSFET experiment by K.v. Klitzing et. al. (K.v. Klitzing, 1980, 1981) the gate voltage V_g is experimentally controlled and the Hall conductance is measured as a function of gate voltage. Since the gate voltage changes the confining potential along the x_3 direction, it is natural to expect that it causes the chemical potential μ to vary. Therefore the experimentally controlled quantity is the chemical potential but not the number density. The number density of course depends on the chemical potential. The number density should therefore be calculated in the grand canonical ensemble. Thus the density operator is introduced:

$$\hat{\rho}_G = \left[\mathrm{Tr} \exp\left(-\frac{1}{k_B T} (H - \mu N) \right) \right]^{-1} \exp\left(-\frac{1}{k_B T} (H - \mu N) \right) \qquad (2.259)$$

to calculate the expectation value of the number density operator in (2.258). The Hamiltonian H is the effective 2D Hamiltonian:

$$H = H_o + H_{spin} + H_{coul} + H_{imp} \qquad (2.260)$$

Where the coupling to the electrical field H_E is omitted on the basis of linear response theory. The total number operator N is:

$$N \equiv \int d\vec{x}\, \hat{\rho}(\vec{x}) \qquad (2.261)$$

Now it will be assumed that the sector of the Hilbert space responsible for the IQHE can be spanned by the scattering states. The validity of this hypothesis can only be tested by comparing the results with experimental data. Localized electron states will not be explicitly accounted for. Therefore the in-Hamiltonian can be written as:

$$H = \sum_n \sum_s \sum_v \sum_\zeta E(n,s,v,\zeta)\, a^{in+}_{nsv\zeta}\, a^{in}_{nsv\zeta} \qquad (2.262)$$

Where the energy spectrum $E(n,s,v,\zeta)$ is a broadened Landau level with the Landau level quantum number n, spin s, the valley v and the remaining quantum number ζ. The asymptotic field satisfies:

$$[a^{in}_{nsv\zeta},\ a^{in+}_{n's'v'\zeta'}] = \delta_{n,n'}\ \delta_{s,s'}\ \delta_{v,v'}\ \delta_{\zeta,\zeta'} \qquad (2.263)$$

It is assumed that the effects of the Coulomb and the impurities interactions are to broaden the unperturbed Landau levels and to change the effective mass m* and the effective g-factor g*. Even in the presence of these interactions the number of states belonging to a single Landau level remains the same:

$$\sum_\zeta = \frac{eB}{hc}\int d\tilde{x} \qquad (2.264)$$

The broadening of the Landau levels is introduced by the number of states density function $g_{nsv}(\varepsilon)$:

{the number of states with energy in between

$E^o_{nsv} + \varepsilon$ and $E^o_{nsv} + \varepsilon + d\varepsilon$}

$$= \frac{eB}{hc}\, g_{nsv}(\varepsilon)\, d\varepsilon \int d\tilde{x} \qquad (2.265)$$

where the energy spectrum is:

$$E^o_{nsv} = \hbar\omega_c\left(n + \frac{1}{2}\right) + \frac{g^*}{2} s\ \mu_B B + vE_v \qquad (2.266)$$

$$s = \pm 1, \quad v = \pm 1$$

Here the effective mass m*, the effective g* and the effective valley

splitting energy E_v have been introduced (K.v. Klitzing, 1981).

$$\omega_c = \frac{eB}{m^*c} \tag{2.267}$$

The fixed number of states belonging to one Landau level (2.264) implies:

$$\int_{-\infty}^{\infty} d\varepsilon \; g_{nsv}(\varepsilon) = 1 \tag{2.268}$$

The effects of the Coulomb and the impurity interactions come into the theory through the effective quantities m^*, g^*, E_v and the function $g_{nsv}(\varepsilon)$.

Now the grand canonical ensemble average for the number density operator in (2.258) can be expressed as:

$$n_e = \frac{eB}{hc} \sum_n \sum_s \sum_v \int_{-\infty}^{\infty} d\varepsilon \; g_{nsv}(\varepsilon) \left[1 + \exp\{\frac{1}{k_B T} (E^0_{nsv} + \varepsilon - \mu)\} \right]^{-1} \tag{2.269}$$

Therefore σ_H (2.258) becomes:

$$\sigma_H(T,B,\mu) = -\frac{e^2}{h} \sum_{nsv} \int_{-\infty}^{\infty} d\varepsilon \; g_{nsv}(\varepsilon) \cdot \left[1 + \exp\left(\frac{1}{k_B T} (E^0_{nsv} + \varepsilon - \mu) \right) \right]^{-1} \tag{2.270}$$

This formula, together with (2.266) and a simple choice for g_{nsv}, will be shown in Chapter 3 to reproduce the experimental results of K.v. Klitzing, 1980) in detail.

In order to obtain analytical results for $\sigma_H(T,B,\mu)$ the following simple model for $g_{nsv}(\varepsilon)$ can be assumed:

$$g_{nsv}(\varepsilon) = \begin{cases} (2\Gamma)^{-1} & \text{if} \quad |\varepsilon| < \Gamma \\ 0 & \text{if} \quad |\varepsilon| > \Gamma \end{cases} \tag{2.271}$$

Of course, any other form for $g_{nsv}(\epsilon)$ can be assumed, such as a Gaussian etc., but this simple form allows the integration in (2.270) to be performed giving:

$$\sigma_H = -\frac{e^2}{h} \sum_{nsv} \left[1 + \frac{k_B T}{2\Gamma} \ln \left[\frac{1 + \exp\{\frac{1}{k_B T}(E^o_{nsv} - \mu + \Gamma)\}}{1 + \exp\{\frac{1}{k_B T}(E^o_{nsv} - \mu - \Gamma)\}} \right] \right] \qquad (2.272)$$

It has been seen in this section that the IQHE can be understood on the basis of formula (2.258) with out considering the mechanism of localization (H. Aoki, 1981) or Laughlin's fundamental gauge principle (R.B. Laughlin, 1981). One may hope that the FQHE can also be explained by (2.270) through the properties of the number of states density function $g_{nsv}(\epsilon)$.

2.4 APPLICATION OF EXACT QFTMB METHODS TO THE FQHE

Whenever physical quantities are calculated using perturbation theory, extra care has to be taken to preserve the symmetries of the model in these approximations.

In section 2.1 it was shown that from the current conservation law one can derive the W-T relation (2.20) which connect the current and density response functions (2.12), (2.13) and (2.15). These relations can then be used to ensure that approximations to the response functions also obey the initial symmetries of the system, i.e. the current conservation. These relations can also be used to deduce the functional form of the response functions without having to evaluate them exactly.

2.4.1 Current Conservation Violation in the Tao-Thouless Model

Attempts to the difficult task of describing the FQHE by microscopic QFTMB models have been few (R. Tao, 1983), (R. Jackiw, 1984a, b, c).

In this section the Tao-Thouless model (R. Tao, 1983) will be studied and some difficulties with it will be pointed out.

In the T-T model the FQHE arises solely from the interplay of the Coulomb interaction between the electrons and the strong external magnetic field. The 2D Hamiltonian of the system is:

$$H = H_o + H_{int} = \int d\tilde{x}\ \hat{\phi}^{+}(\tilde{x}t)\ \frac{1}{2m}\left[- i\hbar\tilde{\nabla} + \frac{e}{c}\tilde{A}\right]^2 \hat{\phi}(\tilde{x}t)$$

$$+ \int d\tilde{x}d\tilde{x}'\ \hat{\phi}^{+}(\tilde{x}t)\ \hat{\phi}^{+}(\tilde{x}'t)\ \frac{e^2}{\varepsilon|\tilde{x} - \tilde{x}'|}\ \hat{\phi}(\tilde{x}'t)\ \hat{\phi}(\tilde{x}t) \qquad (2.273)$$

where ε is the dielectric constant, and only one direction of spin is considered. The gauge for the vector field is chosen such that

$$\tilde{A}(\tilde{x}) = (- Bx_2, 0) \qquad (2.274)$$

then the n^{th} Landau level single electron wave function is:

$$u_{n,s}(\tilde{x}) = (\ell L \sqrt{\pi}\ n!\ 2^n)^{-1/2}\ \exp\left[ikx_1 - \frac{1}{2}\left(\frac{x_2}{\ell} - k\ell\right)^2\right]$$

$$H_n\left(\frac{x_2 - k\ell^2}{\ell}\right) \qquad (2.275)$$

where L^2 is the area of the system, H_n is the Hermite polynomial and:

$$\ell^2 = \frac{\hbar c}{eB}\ , \quad k = \frac{2\pi s}{L}\ , \quad (s = 0, \pm 1, \pm 2 \ldots) \qquad (2.276)$$

The Heisenberg field operator $\hat{\phi}(\tilde{x}t)$ can be expanded as:

$$\hat{\phi}(\underset{\sim}{x}t) = \sum_{n=0}^{\infty} \sum_{s=-N/2}^{N/2} \hat{c}_{ns}(t)\, u_{ns}(\underset{\sim}{x}), \quad N = \frac{L^2}{2\pi\ell^2} \tag{2.277}$$

Tao and Thouless then made the approximation that due to the very strong external magnetic field, it is enough to consider only the lowest Landau level (n = 0) in the expansion (2.276) for the field operator. They also made the assumption that the groundstate $|\phi_o(p)\rangle$, in the interaction picture, is a superlattice in the lowest Landau orbital space (characterized by the parameter s or k) such that every p^{th} s-site is occupied by an electron. The filling factor ν is therefore given by:

$$\nu = \frac{1}{p} \quad (P = \text{integer}) \tag{2.278}$$

In this groundstate $|\phi_o(p)\rangle$ the particles and holes are separated by an energy gap Δ which is supposed to be caused by the Coulomb interaction.

The free Greens function in the interaction picture is given by:

$$iG_o(\underset{\sim}{x}t,\underset{\sim}{x}'t') \equiv \langle\phi_o(p)|T\, \hat{\phi}^{in}(\underset{\sim}{x}\, t\,)\, \hat{\phi}^{+in}(\underset{\sim}{x}'t')\, |\phi_o(p)\rangle$$

$$\equiv \frac{1}{2\pi}\int d\omega\, e^{-i\omega(t-t')}\, iGo(\underset{\sim}{x},\underset{\sim}{x}',\omega) \tag{2.279}$$

with:

$$G_o(\underset{\sim}{x},\underset{\sim}{x}',\omega) = \sum_s u_{os}(\underset{\sim}{x})\, u^*_{os}(\underset{\sim}{x}')\left[\frac{1 - f(s)}{\omega - \varepsilon^+ + i\eta} + \frac{f(s)}{\omega - \varepsilon^- - i\eta}\right] \tag{2.280}$$

where the distribution function f(s) reflects the superlattice structure of $|\phi_o(p)\rangle$:

$$f(s) \equiv \langle\phi_o(p)|\, \hat{c}^+_{os}\, \hat{c}_{os}\, |\phi_o(p)\rangle = \sum_{m=-N/2P}^{N/2P} \delta_{s,mp} \tag{2.281}$$

and ε^+ and ε^- are given by:

$$\varepsilon^{+} = \omega_c + \frac{1}{2}\Delta$$
$$\varepsilon^{-} = \omega_c - \frac{1}{2}\Delta \qquad (2.282)$$

where the gap parameter Δ is supposed to be found in a self consistent way, i.e. Tao and Thouless calculated the self energy correction to the Greens function according to the RPA using G^o and thereby determined Δ self consistently. Δ then depends on the filling factor $\nu = \frac{1}{p}$ and is largest for $\nu = \frac{1}{2}$. The T-T model offers no mechanism to suppress even denominal filling factors, which are not observed in experiments (H.L. Störmer, 1984).

Now the Causal density correlation function can also be calculated from the self consistent Greens function (2.278).

In the lowest order:

$$iD^c_{oo}(\tilde{x}t,\tilde{x}'t') \equiv e^2 \langle \phi_o(p)|T\, \hat{\phi}^{in}(\tilde{x}t)\, \hat{\phi}^{in}(\tilde{x}t)\, \hat{\phi}^{in^+}(\tilde{x}'t')$$
$$\hat{\phi}^{in}(\tilde{x}'t')|\phi_o(p)\rangle$$
$$- e^2 n_e^2$$
$$= e^2 G_o(\tilde{x}t,\tilde{x}'t')\, G_o(\tilde{x}'t',\tilde{x}t) \qquad (2.283)$$

If the discrete summation $\sum_s$ is replaced by the continuous integral $\int ds$, then the density correlation function turns out to be a function of $\tilde{x} - \tilde{x}'$ so that its Fourier transform can be introduced as:

$$D^c_{oo}(\tilde{x} - \tilde{x}', t - t') = \frac{1}{(2\pi)^3}\int d\tilde{q}d\omega\, e^{i\tilde{q}\cdot(\tilde{x} - \tilde{x}') - i\omega(t - t')} \cdot D^c_{oo}(\tilde{q},\omega) \qquad (2.284a)$$

The wavefunctions defined by (2.274) are of the same form as (2.137), therefore $D^c_{oo}(\tilde{q},\omega)$ can be evaluated using the integrals (2.149) and (2.150), resulting in:

$$D^c_{oo}(\tilde{q},\omega) = \frac{1}{2\pi\ell^2}\,\nu(1-\nu)\,e^{-\frac{\ell^2 q^2}{2}} \cdot \left(\frac{1}{\omega - \Delta + i\eta} - \frac{1}{\omega + \Delta - i\eta}\right)_{\eta\to 0^+} \tag{2.284b}$$

The retarded correlation function, or the response function, therefore has the form:

$$D^R_{oo}(\tilde{q},\omega) = \frac{1}{2\pi\ell^2}\,\nu(1-\nu)\,e^{-\frac{\ell^2 q^2}{2}} \cdot \left(\frac{1}{\omega - \Delta + i\eta} - \frac{1}{\omega + \Delta + i\eta}\right) \tag{2.285}$$

This result (2.285) immediately gives the effective potential as well as the generalized dielectric function $k(\tilde{q},\omega)$ in the RPA:

$$V_{eff}(\tilde{q},\omega) = \frac{2\pi e^2}{\varepsilon\left[\tilde{q} - \frac{2\pi e^2}{\hbar\varepsilon} D^R_{oo}(\tilde{q},\omega)\right]} \equiv \frac{1}{K(q,\omega)}\,\frac{2\pi e^2}{q} \tag{2.286}$$

where:

$$K(q,\omega) = \varepsilon - \frac{2\pi e^2}{\hbar q}\,\frac{1}{2\pi\ell^2}\,\nu(1-\nu)\,e^{-\frac{\ell^2 q^2}{2}} \cdot \left(\frac{1}{\omega - \Delta - i\eta} - \frac{1}{\omega + \Delta + i\eta}\right) \tag{2.287}$$

From which the dispersion relation of the longitudinal plasmon becomes:

$$\omega^2(q) = \Delta^2 + \frac{e^2}{\varepsilon \ell^2 \hbar q}\, 2\Delta\nu(1-\nu)\exp\left[-\frac{q^2\ell^2}{2}\right] \qquad (2.288)$$

The second term on the r.h.s. diverges as $q \to 0$ so it is necessary to find out whether this divergence is caused by the approximations used, i.e. the truncation of the Hilbert space etc., or if it reflects properties of the initial model.

In section 2.1.2 the f-sum rule, for a system with a groundstate of uniform density, was derived from the W-T-relations (2.20). For the exact $D^R_{oo}(k,\omega)$ the f-sum rule gives:

$$\int_0^\infty \omega d\omega\ \mathrm{Im}\ D^R_{oo}(q,\omega) = -\frac{\pi\hbar n_e e^2}{2m} q^2 \qquad (2.289)$$

On the other hand the same expression for the approximate response function (2.285) gives:

$$\int_0^\infty \omega d\omega\ \mathrm{Im}\ D^R_{oo}(q,\omega) = -\pi\nu(1-\nu)\frac{\Delta}{2\pi\ell^2}\, e^{-\frac{q^2\ell^2}{2}} \qquad (2.290)$$

since only the lowest Landau level was used to derive $D^R_{oo}(q,\omega)$ in the T-T model. It can be shown that the lowest order response function, derived in an untruncated Hilbert space satisfies (2.289) if $\Delta \neq 0$.

It is interesting to note that if Δ in eq. (2.290) is chosen such as to give the correct f-sum rule (2.289), i.e. Δ will be used to account for corrections due to higher Landau levels instead of being caused by the Coulomb interaction, then:

$$\Delta = \frac{\hbar q^2}{2m(1-\nu)} \exp\left(\frac{\ell^2 q^2}{2}\right) \equiv \Delta(q) \qquad (2.291)$$

which implies q dependence. Here the relation

$$n_e = \langle \Psi_o | \hat{\phi}^+(\tilde{x}t)\ \hat{\phi}(\tilde{x}t) | \Psi_o \rangle = \frac{\nu}{2\pi\ell^2} \tag{2.292}$$

was used to derive (2.291) from (2.289) and (2.290). If now this $\Delta(q)$ (2.291) is used in the dispersion relation (2.288) then:

$$\omega^2(q^2) = \Delta^2(q^2) + \frac{2\pi e^2 n_e}{m} q \tag{2.293}$$

where the apparently singular term has turned into the leading term of the dispersion relation for a 2D electron gas in no external magnetic field. The important questions now is why does the use of the f-sum rule get rid of the singularity in eq. (2.288).

To answer this question it will be shown that the Greens function assumed by Tao and Thouless (2.279) violates the current conservation in the model. The f-sum rule was derived from the W-T relations (2.20) which reflect the conservation of the current. Therefore the f-sum rule imposes the current conservation on the density response function.

The current of the T-T model in the Heisenberg picture is:

$$\hat{j}_\nu(\tilde{x}t) = \frac{ie\hbar}{2m} \hat{\phi}^+(\tilde{x}t)(\partial_\nu - \overleftarrow{\partial}_\nu)\hat{\phi}(\tilde{x}t) - \frac{e^2}{mc} A_\nu(\tilde{x})\ \hat{\phi}^+(\tilde{x}t)\ \hat{\phi}(\tilde{x}t) \tag{2.294}$$

and the density is:

$$\hat{\rho}(\tilde{x}t) = -\ e\ \hat{\phi}^+(\tilde{x}t)\ \hat{\phi}(\tilde{x}t) \tag{2.295}$$

The Greens function (2.279) implies the following asymptotic field:

$$\hat{\phi}^{in}(\tilde{x}t) = \sum_{s \neq mp} a_s u_{os}(\tilde{x})\ e^{-i\epsilon^+ t} + \sum_{s'=mp} b^+_{s'} u_{os'}(\tilde{x})\ e^{i\epsilon^- t} \tag{2.296}$$

If the current $\hat{j}^{in}$ is assumed to have the same functional form as (2.294), and the density as (2.295), then one can show that the current is purely transverse, a peculiar property of the lowest order Landau subspace:

$$\partial_1 \hat{j}_1^{in}(\tilde{x}t) + \partial_2 \hat{j}_2^{in}(\tilde{x}t) = 0 \tag{2.297}$$

This is caused by the following property of the wavefunctions u_{os}:

$$\partial_1 u_{os}(\tilde{x}) = ik\, u_{os}(\tilde{x})$$

$$\partial_2 u_{os}(\tilde{x}) = -\frac{1}{\ell}\left(\frac{x_2}{\ell} - k\ell\right) u_{os}(\tilde{x}) \tag{2.298}$$

It is important to note here that the transversality of the current does not mean the vanishing of the longitudinal plasmon mode, since the current is not conserved.

The time derivative of the density operator can be found by using (2.296) and (2.295)

$$\partial_t \hat{\rho}^{in}(\tilde{x}t) = -ie(\varepsilon^+ - \varepsilon^-) \sum_{ss'} \left[a_s^+ b_{s'}^+, e^{it(\varepsilon^+ - \varepsilon^-)} + h.c. \right] \tag{2.299}$$

and therefore:

$$\partial_t \hat{\rho}^{in}(\tilde{x}t) + \sum_{\nu=1}^{2} \partial_\nu \hat{j}^{in}(\tilde{x}t) = -ie(\varepsilon^+ - \varepsilon^-) \sum_{ss} \{a_s^+ b_s^+, e^{it(\varepsilon^+ - \varepsilon^-)} + h.c.\} \neq 0 \tag{2.300}$$

Therefore the current $\hat{j}^{in}$, implied by the Greens function (2.279), is not conserved due to the energy gap $\Delta = \varepsilon^+ - \varepsilon^-$.

The T-T model is an important step in the direction of explaining the FQHE by QFTMB methods, and as such more research is needed on it. Definitely the Hilbert space should not be truncated to the lowest Landau level as the f-sum rules (2.289) and (2.290) show. Secondly the question

of whether the superlattice groundstate is really the proper groundstate of the Coulomb interacting electrons has not been answered yet. And in this connection one must have in mind the effective Coulomb interaction introduced in (2.235).

The self consistent determination of the gap parameter Δ, caused by the interplay of the Coulomb interaction and the external magnetic field, has also to be looked carefully into. In this connection one may also ask whether the current for the asymptotic field might have to be defined on the basis of the equation of motion so that it will be conserved.

The accuracy of the fractional quantization of the Hall conductivity also makes doubtful the commonly held view that the FQHE is caused by the Coulomb interactions.

2.4.2 The Exact Hamiltonian for Laughlin's Wavefunctions

R.B. Laughlin has proposed variational groundstate wavefunctions, which he claims to describe the condensation of a 2D electron gas into a new state of matter an incompressible quantum fluid (R.B. Laughlin 1983b, 1984). The details of how these wavefunctions describe the FQHE will not be entered into here, instead the confusion about what is the exact Hamiltonian of these wavefunctions will be cleared up. S.M. Girvin and T. Jach (S.M. Girvin, 1983) claim that the Laughlin wavefunctions are exact solutions to the problem of harmonically interacting electrons. This claim will be shown to be wrong, especially since they used an approximation, replacing the Landau level mixing operators by their expectation values, in order to justify their claim.

The Laughlin wavefunction describing the FQHE at the filling factor

$\nu = 1/M$, where M is a positive odd integer, are of the form:

$$\Psi(z_0, z_1 \ldots . z_n) = \prod_{i>j} (z_i - z_j)^M \exp\left[- \frac{1}{4\ell^2} \sum_i |z_i|^2 \right] \tag{2.301}$$

where the co-ordinates of the n+1 electrons are described by:

$$z_j = x_j + iy_j \qquad (j = 0.1 \ldots n) \tag{2.302}$$

and as before $$\ell^2 = \frac{\hbar c}{eB} \tag{2.303}$$

These wavefunctions (2.301) are considered as trial wavefunctions for the Hamiltonian

$$H = \sum_{j=0}^{n} \left[\frac{1}{2m} \left(\frac{\hbar}{i} \tilde{\nabla}_j + \frac{e}{c} \tilde{A}_j \right)^2 + V(z_j) \right] + \sum_{j>k} \frac{e^2}{|z_j - z_k|} \tag{2.304}$$

where $V(z_j)$ is a potential generated by a uniform neutralizing background. $\tilde{A}_j$ is the vector potential causing the external constant magnetic field in the direction perpendicular to the electron plane.

$$\tilde{A}_j = \left(- \frac{1}{2} yB, \frac{1}{2} xB \right) \tag{2.305}$$

The eigenfunctions of the free Hamiltonian

$$H_{oj} = \frac{1}{2m} \sum_{j=1}^{n} \left[\frac{\hbar}{i} \tilde{\nabla}_j + \frac{e}{c} \tilde{A}_j \right]^2 \tag{2.306}$$

are of the form:

$$\phi_m(z_j) = (2^{m+1} \pi m!)^{-1/2} z_j^m \exp\left[- \frac{1}{4\ell^2} |z_j|^2 \right] \tag{2.307}$$

where $m = 0, 1, \ldots . \infty$. A Slater determinant of n+1 of these wavefunctions has the form:

$$\psi(z_0 \ldots . z_n) = \begin{bmatrix} \phi_0(z_1) & \phi_0(z_2) \ldots \phi_0(z_n) \\ \vdots & \\ \phi_n(z_1) & \phi_n(z_2) \ldots \phi_n(z_n) \end{bmatrix}$$

$$\sim \begin{bmatrix} z_0^0 & z_1^0 & \ldots & z_n^0 \\ z_0^1 & z_1^1 & \ldots & z_n^1 \\ z_0^2 & z_1^2 & & \vdots \\ \vdots & \vdots & & \\ z_0^n & z_1^n & \ldots & z_n^n \end{bmatrix} \cdot \exp\left[- \frac{1}{4\ell^2} \sum_j |z_j|^2 \right]$$

$$= \prod_{i>j} (z_i - z_j) \exp\left[- \frac{1}{4\ell^2} \sum_j |z_j|^2 \right] \qquad (2.308)$$

where the last equality is obtained by identifying the determinant, in the line before, as the Vandermond determinant.

Due to (2.308) it is possible to write the Laughlin wavefunction (2.301) as a power of the Slater determinant of the wavefunctions for the Hamiltonian (2.306) if the electrons had the fractional charge - e/M.

$$\Psi(z_0 \ldots . z_n) = \left[\psi(z_0 \ldots . z_n, - \frac{e}{M}) \right]^M \qquad (2.309)$$

This possibility has not been pointed out in the literature yet, but it shows that the fractional charge content of (2.301) is almost trivial.

Now the exact Hamiltonian for (2.301) can be found by applying the free Hamiltonian

$$H_o = \sum_i H_{oi} \qquad (2.310)$$

with

$$H_{oi} = \frac{1}{2m} (-i\hbar \nabla_i + \frac{e}{c} \tilde{A}(\tilde{x}))^2 \qquad (2.311)$$

on the wavefunction (2.309)

$$H_{oi}\Psi = -\frac{\hbar^2}{2m}\left[\left(\partial_{x_i} - \frac{iy_i}{\ell^2}\right)^2 + \left(\partial_{y_i} + \frac{ix_i}{\ell^2}\right)^2\right]\Psi$$

$$= -\frac{\hbar^2}{2m}\left[\tilde{\nabla}^2 + \frac{2i}{\ell^2}\left(x_i\,\partial_{x_i} - y_i\,\partial_{y_i}\right) - \frac{|z_i|^2}{\ell^4}\right]\Psi . \qquad (2.312)$$

By using the co-ordinate change:

$$x = \frac{1}{2}(z + z^*)$$
$$y = \frac{1}{2i}(z - z^*) \qquad (2.313)$$

leading to:

$$\tilde{\nabla}^2 = 4\,\frac{\partial^2}{\partial z \partial z^*} \qquad (2.314)$$

and:

$$x\,\frac{\partial}{\partial y} - y\,\frac{\partial}{\partial x} = i\left(z\,\frac{\partial}{\partial z} - z^*\,\frac{\partial}{\partial z^*}\right) \qquad (2.315)$$

and carrying out the pending derivatives, then eq. (2.312) becomes, once the sum over i is performed.

$$H_o\,\Psi = \frac{3\hbar^2}{2m}\,\Psi \sum_i \left[\frac{|z_i|^2}{4\ell^2} + \frac{M}{\ell^2}\sum_{j\neq i}\frac{z_i}{z_i - z_j}\right] \qquad (2.316)$$

The summations of the second term on the r.h.s. can be carried out giving:

$$\sum_{\substack{i,j=0 \\ j\neq i}}^{n} \frac{z_i}{z_i - z_j} = \frac{(n+1)n}{2} \qquad (2.317)$$

Therefore a Laughlin wavefunction of n electrons

$$\Psi(\tilde{x}_1 \ldots . \tilde{x}_n) = \prod_{i>j}(z_i - z_j)^M \exp\left[-\frac{1}{4\ell^2}\sum_i |z_i|^2\right] \qquad (2.318)$$

is an exact eigen-function of the Hamiltonian

$$H = \sum_i \left(\frac{-1}{2m} \left(-i\hbar \tilde{\nabla}_i + \frac{e}{c} \tilde{A}(\tilde{x})\right)^2 - \frac{3\hbar^2}{8m\ell^4} |\tilde{x}_i|^2 \right) \tag{2.319}$$

with eigen-energy:

$$\frac{3\hbar^2}{4m\ell^2} Mn(n-1) = \frac{3}{4} \hbar\omega_c \, Mn(n-1) \tag{2.320}$$

and

$$\tilde{A}(\tilde{x}) = \frac{1}{2} B(-y,x) \, . \tag{2.321}$$

This Hamiltonian derived without an approximation does not describe harmonically interacting electrons, there is no coupling between the electrons.

There are many unanswered questions about the Laughlin wavefunctions; For example what is the meaning of eq. (2.309), can it be used as a starting point for a QFTMB description of Laughlin's model? Is there a connection between these wavefunctions and the Tao-Thouless groundstate (R. Tao, 1983)?

CHAPTER 3

Experimental Results for the QHE

The aim of this chapter is to show that the simple but general model of the IQHE, introduced in section 2.3, can reproduce in detail the experimental results by K.v. Klitzing (K.v. Klitzing, 1980, 1981). These experiments were carried out in a Si-MOSFET, while later experiments have more commonly been performed in GaAs heterostructures (see sec. 1.5.4). The chapter therefore commences by a description of these two experimental systems emphasizing their important differences and similarities. The chapter then continues with a description of some of the experimental results for the QHEs emphasizing the role of the chemical potential as the relevant thermodynamic variable rather than the electron number density. It will then be shown how closely the microscopic model of section 2.3 can fit the experimental results of K.v. Klitzing.

3.1 DESCRIPTION OF EXPERIMENTAL SYSTEMS

The QHE's were found in two quite different experimental systems, the Si-MOSFET and the GaAs heterostructure. As will be seen later, it is this difference which will lead to the fuller understanding of the QHE. First these systems have to be described starting with the Si-MOSFET.

MOSFET

The structure of a MOSFET is shown in Fig. 3. The metal gate is insulated from the p-type bulk Si. by a thin SiO_2 layer. There is a potential step of 3.15 eV between the p-Si bulk and the SiO_2 insulator. A current can flow between the source and drain contacts only if a positive

bias V_g is applied to the gate in order to produce an n-type layer (channel) on the surface of p-type Si adjacent to the SiO_2 insulator. The band structure of the MOSFET can be seen in Fig. 4. When a positive bias is applied to the gate V_g, a negative charge will be induced in the bulk Si by removing holes from its valence band, creating the so called depletion layer (0.1-10 μm). If the positive bias V_g becomes very strong then the conduction band can be bent below the Fermi level and electrons will be induced in the bulk Si at the interface. When the surface density of electrons is higher than the equivalent hole density of the bulk then this thin (10-100A°) (C.C. Grimes, 1978) layer is called inversion layer. The realizable 2D electron density of the inversion layer is $10^{10} \leq n_e \leq 10^{13} cm^{-2}$ (T. Ando, 1982a, p.442) (S. Kawaji, 1983). This density is controlled by the gate voltage V_g, and can be measured by the Shubnikov-de Haas effect (see section 1.2.4). The motion of the electrons perpendicular to the Si-SiO_2 interface is quantized into discrete energy levels (subbands) of the confining potential. If $n_e \sim 10^{12} cm^{-2}$, the dielectric constant $\varepsilon = 11.5$ and the concentration of acceptors $N_A = 1.65 \cdot 10^{15} cm^{-3}$ then the energy levels in the Si(100) n-type inversion layer are (T. Ando, 1975b):

$$E_F - E_0 \sim 5 \text{ meV}$$

$$E_1 - E_0 \sim 21 \text{ meV}$$

$$E_2 - E_0 \sim 36 \text{ meV}$$

$$E_3 - E_0 \sim 46 \text{ meV}$$

so obviously under these conditions, and at a low temperature, only the lowest subband is populated.

Si has 6 conduction band minimas (valleys) in the <100> direction

of momentum space (N.W. Ashcroft, 1976). The ground state of the electrons has two valleys with effective mass $m_\ell^* = 0.916m$ in the direction perpendicular to the interface. It has been observed in experiments (A.B. Fowler, 1966) that this degeneracy is lifted though the mechanism is not well understood. (F.J. Okawa, 1976, 1977a, b) (L.J. Sham, 1978, 1979) (R. Kümmel, 1975). The effective mass for the motion parallel to the interface is $m_t^* = 0.190$ m.

The mobility μ of electrons in Si inversion layers is up to $10^4 cm^2/Vs$ (S. Kawaji, 1983). It is measured from the cyclotron line width or the Shubnikov-de Haas amplitude (see section 1.2.4).

GaAs

The GaAs heterostructure, Fig. 5, is quite different from the Si-MOSFET; the 2D electrons reside in a natural discontinuity of the conduction bands at the AL_x-$Ga_{1-x}As$ - GaAs interface, see Fig. 6 for the band structure. (The discontinuity is about 300 meV (G.A. Baraff, 1981)). The crystal lattices are matched at the interface, in contrast to the interface of a glassy insulator and a crystalline Si in a MOSFET. The GaAs layer in which the 2D electron gas is located is undoped and the $AL_xGa_{1-x}As$ is modulation doped (H.L. Störmer, 1984) such that closest to the interface there is a thin ($\sim$50A°) undoped layer. This results in the GaAs heterostructure having mobilities much higher than MOSFET's, i.e. $\mu \leq 10^6$ cm^2/Vs (S. Kawaji, 1983) equivalent to a mean free path $\lambda \sim 5\mu m$ (H.L. Störmer, 1984).

The electrons are supplied by the Si donors in the $AL_xGa_{1-x}As$ layer, and in most experiments (H.L. Störmer, 1983) (D.C. Tsui, 1981a) their 2D density is varied by the doping to stay in the range $n_e \sim 1.4\text{-}4.2 \cdot 10^{11}$

cm^{-2} when T = 0.5-4.2K. In some experiments the GaAs heterostructure is made into a FET (Field Effect Transistor), by adding a substrate and a gate plate, in order to control n_e by V_g (D.C. Tsui, 1981c) (A.M. Chang, 1983). The effective mass m* and the effective g-factor g* in GaAs are m* = 0.068m g* = 0.522 (D.C. Tsui, 1981a).

Probably the purest q-2D electron system known yet is the thin layer of electrons held to the surface of liq. He. The characteristic longitudinal plasmon of a 2D electron gas was first found in this system (C.C. Grimes, 1976).

3.2 QHE EXPERIMENTAL RESULTS

The general results from experiments on the IQHE and the FQHE were reviewed in sections 1.5.4. and 1.5.6. of the first chapter. In this section the results of some few experiments will be presented.

There are two types of experiments; In experiments performed on MOSFETS, see the results of K.v. Klitzing et. al. (K.v. Klitzing, 1981) in Fig. 7, the resistivities $\rho_{xy} = 1/\sigma_H$ and ρ_{xx} are measured as functions of the gate voltage V_g, since increased V_g means increased density and therefore the population of higher and higher Landau levels. The spin and valley splitting is clearly seen for the lowest Landau levels. In this experiment the magnetic field is held constant at 18.9T. In the experiments using GaAs heterostructures, which usually have no gate, the magnetic field B is varied from 0 upto 30T, see Fig. 8 for experimental results of M.A. Paalanen et. al. (M.A. Paalanen, 1982), and Fig. 9 for the experimental results of K.v. Klitzing et. al. (K.v. Klitzing, 1982). In these experiments successive Landau levels are depopulated by increasing B, since the 2D number density of electrons each Landau level can

hold is:

$$n_e = \frac{eB}{hC} \tag{3.1}$$

and the total density n_e can only vary within some limits (see below). The spin splitting is resolved for the lowest Landau levels. In both types of experiments the sharp plateaus are seen in ρ_{xy} with the simultaneous vanishing of ρ_{xx} at low enough temperatures. The plateaus are generally wider in the GaAs heterostructure than in the MOSFET.

The results for the FQHE are shown in Fig. 10 from the experiment of H.L. Störmer et. al. (H.L. Störmer, 1983), and in Fig. 11 from the experiment of K.v. Klitzing et. al. (K.v. Klitzing, 1984). These experiments are carried out on GaAs heterostructures of high mobility and at low temperature. The temperature dependence of the FQHE is shown in Fig. 12 from the experiment of D.C. Tsui et. al. (D.C. Tsui, 1982). The dip in ρ_{xx} develops before the plateau in ρ_{xy} in the FQHE as the temperature is lowered. It is obvious from both types of experiments that the 2D electron density n_e is not constant in the samples since σ_H develops flat plateaus, both as a function of V_g and B. Thus there has to be some reservoir of electrons in the samples (G.A. Baraff, 1981). In the GaAs-heterostructure the Si-donors in the $AL_xGa_{1-x}As$ layer act as a reservoir, while in MOSFET's the reservoir mechanism is not well understood (G.A. Baraff, 1981). If in MOSFET's there were no reservoirs then the electron density in the inversion layer would depend linearly on the gate voltage V_g according to Gauss's Law, resulting in no plateaus to be seen in σ_H. But since there is a reservoir an increase in the gate voltage does not have to lead to a directly proportional increase in n_e, some electrons can be transferred between the inversion layer and the reservoir. There-

fore the chemical potential μ has to be considered the appropriate thermodynamical variable instead of the density n_e. The change in the gate voltage in experiments on MOSFET's has therefore to be related to a change in the chemical potential μ.

3.3 MODEL REPRODUCTION OF EXPERIMENTAL RESULTS

In section 2.3 the following expression was derived for the Hall conductivity (2.270):

$$\sigma_H(T,B,\mu) = -\frac{e^2}{h} \sum_{nsv} \int_{-\infty}^{\infty} d\varepsilon g_{nsv}(\varepsilon) \left[1 + \exp\left[\frac{1}{k_B T}(E^o_{nsv} + \varepsilon - \mu)\right]\right]^{-1} \tag{3.2}$$

where g_{nsv} is the number of states density function introducted in (2.265) and E^o_{nsv} is the energy spectrum.

$$E^o_{nsv} = \hbar\omega_c(n + 1/2) + \frac{g^*}{2} s\mu_B B + vE_v \tag{3.3}$$

If the lineshape (the number of states density function) is assumed to be Gaussian:

$$g_{nsv}(\varepsilon) = \frac{1}{\sqrt{2\pi\Gamma^2}} \exp(-\frac{\varepsilon^2}{2\Gamma}) \tag{3.4}$$

where Γ represents the line width, then the integration in (3.2) can be performed numerically. The results are shown in Figures 13-15, where σ_H is plotted as a function of the chemical potential μ, s and v are chosen to be zero and $\hbar\omega_c$ = 10 meV while Γ is 0.5 meV in Fig. 13, 1.0 meV in Fig. 14 and 2.0 meV in Fig. 15. In each figure the curve is shown

for the two temperatures T = 0.58K and T = 5.8K. Figures 13 and 14 show clear plateaus with rounded off corners at the higher temperature, while in Figure 15 no plateaus are seen due to the large linewidth. Similar curves can be obtained for σ_H as a function of B.

If the simpler linewidth is chosen (2.271):

$$g_{nsv}(\varepsilon) = \begin{cases} (2\Gamma)^{-1} & \text{if } |\varepsilon|<\Gamma \\ 0 & \text{if } |\varepsilon|>\Gamma \end{cases} \tag{3.5}$$

then the integral in (2.270) can be evaluated analytically giving (2.272):

$$\sigma_H = -\frac{e^2}{h} \sum_{nsv} \left[1 + \frac{k_B T}{2\Gamma} \ln \left[\frac{1 + \exp\left[\frac{1}{k_B T}(E^o_{nsv} - \mu + \Gamma)\right]}{1 + \exp\left[\frac{1}{k_B T}(E^o_{nsv} - \mu - \Gamma)\right]} \right] \right]$$

which is easier to use to reproduce experimental results. In order to do that for experiments performed on MOSFET's the relation between the gate voltage V_g and the chemical potential μ has to be determined (see section 3.2).

From the experimental results of K.v. Klitzing (K.v. Klitzing, 1980, 1981) in Fig. 7 it is seen that for the range $0<V_g<30V$ the dips in ρ_{xx} are equispaced, indicating that the Landau levels are also equispaced with respect to V_g. One can therefore assume from the μ-dependence of eq. (3.2) that there should be a linear relation between V_g and μ for this range of V_g.

$$\mu = \mu_o + \alpha e\, V_g \tag{3.7}$$

where the parameters μ_o and α can be determined experimentally. With the

choice of parameters:

$$g^* = 5.0,\ E_v = 1.2\ (meV),\ m^* = 0.19m,\ \Gamma = 0.8\ (meV)$$

$$\mu_o = 0,\ \alpha = 9.3 \cdot 10^{-3}\ meV/eV \qquad (3.8)'$$

a very nice fit to the experiment of K.v. Klitzing (K.v. Klitzing, 1981) can be obtained, Fig. 16, where the solid curve represents the model results and the dotted curve shows the experimental results. The choice (3.8) of parameters does by no means represent best fit method. The parameters have been chosen to be close to known experimental values (K.v. Klitzing, 1980, 1981) (T. Ando, 1982a). The linewidth (3.5) is of course an over simplification since g^* and E_v are known to depend on V_g and n_e (Th. Englert, 1978) and to be strongly enhanced by many body effects when only few Landau levels (1 or 2) are populated.

The temperature dependence of σ_H for the same parameters (3.8) is shown in Figures 17 to 23. At T = 0.5K the plateaus are sharp, Fig. 17, while at T = 6.0K the plateaus have disappeared and the curve is very close to the classical results. This model can therefore both account for the IQHE and the CHE. The vanishing of the longtudinal resistivity ρ_{xx} can be understood directly from the energy level structure (3.3) as follows: if the chemical potential μ lies between two adjacent bands, for example between E_{o++} and E_{o+-}, then the bands below the chemical potential are fully occupied and the electrons in those bands cannot be scattered by the impurities unless the supplied energy is sufficient for the electrons to jump into empty bands.

Having successively reproduced the experimental results for the MOSFET experiments of the IQHE, what about the results from experiments on GaAs? In these experiments the magnetic field B is varied instead of the gate

voltage V_g and as in MOSFET's, the density n_e cannot be constant (G.A. Baraff, 1981) (T. Toyoda, 1984b) so the chemical potential has to be treated as the relevant thermodynamical variable. Otherwise eq. (3.2) can not give plateaus in σ_H as a function of B. However the main problem is that g* and the linewidth depend strongly on B, therefore a detailed investigation is required to specify them as functions of B before a good fit can be obtained.

An interesting future task would also be to include higher order effects (interaction effects) in the evaluation of $n_e = \langle\hat{\rho}\rangle$. These higher order effects might explain the small dips in σ_H just in front of a plateau (K.v. Klitzing, 1981), see Figures 7 and 16.

It is also interesting to notice that the FQHE could be explained through the structure of $g_{nsv}(\varepsilon)$ in eq. (3.2). If for example it is found that the lowest Landau level (or higher ones) has tiny energy gaps for high magnetic field at low temperature, for some now unknown reason, then these would cause fractional plateaus in eq. (3.2).

In this chapter it has been shown that the IQHE is well described by the microscopic model introduced in sec. 2.3, together with the assumption that the chemical potential depends linearly on the gate voltage in the region of interest.

CHAPTER 4

CONCLUSION

This thesis has been concerned with some important dynamical properties of q-2D electron systems, such as the possible propagation of transverse electromagnetic waves and the quantum Hall effects. Several new results have been obtained and new methods used in attacking the problem. These will be summarized in this section and some important questions for future research will be raised.

From the outset the point of view has been taken that the finite thickness of q-2D electron systems has to be accounted for, especially when the electrons are interacting with a 3D electromagnetic field. This is accomplished by writing the 3D field operator as a product of the 2D field operator of the electrons in the $(x_1\text{-}x_2)$-plane, and the boundstate wave functions in the x_3-direction (2.9). Then the effective 2D Hamiltonian for the q-2D electron gas can be derived from the full 3D Hamiltonian of the system. Before any approximate calculations of the response functions for the model were performed, the W-T relations connecting the current and density response functions were derived from the current conservation. These relations (2.20) can be used to ensure that approximations to the response functions reflect the current conservation.

For the first time the dynamical current response functions are calculated at T=0 with or without an external magnetic field (2.90-2.101) (2.163-2.167). These response functions are separated into their transverse and longitudinal part. The transverse part is then used in coupling the linear response of the current self consistently to the c-number electromagnetic field, in order to derive the dispersion relations for the

transverse plasmon in q-2D electron systems (2.77). This self consistent linear response method is considerably simpler than the methods employed by D. Bohm and D. Pines (D. Bohm, 1951) or by H. Matsumoto et. al. (H. Matsumoto, 1980) to calculate the 3D plasmon dispersion. In appendix A it is shown that the self consisent linear response method gives the same result as these other methods for the 3D transverse plasmon, where the calculation includes the first order quantum correction.

The 2D transverse plasmons have not been derived before. Remarkably they have a gap which depends on the thickness of the system (2.131) (2.195), thus emphasizing the necessity of the dimensional reduction scheme described earlier.

For future tasks involving the 2D transverse plasmon the following few points can be considered:

Due to the severe restriction placed on the relevant parameters in order to obtain analytic dispersion relations, it is necessary to seek for solutions numerically in regions where the analytic approximations do not hold.

In the derivation of the dispersion relations it is assumed that all the electrons are in the lowest level of the confining potential. This assumption is adequate for low enough temperature, but it might be interesting to see the effects of more than one level being populated. In that connection finite temperature corrections might also be of interest.

The transverse response functions may also be important in connection with 2D superconductors where they relate the super currents and the external magnetic field in the Meissner effects (H. Takagi, 1982a, b, 1983).

In section 2.3 on the IQHE it is shown that the important formula (2.258):

$$\sigma_H = - \frac{n_e ec}{B} \qquad (4.1)$$

holds exactly independent of ρ_{xx} or σ_{xx}. This formula is derived for a Coloumb interacting quantum q-2D electron system in the presence of impurities and at finite temperature. Here the Hall effect condition $I_y = 0$ was used, and the only restriction on the damping contribution to the equation of motion for I_y is that it must vanish when $I_y = 0$. Since the number density n_e of electrons is not constant in experiments, due to the electron reservoirs, the grand canonical ensemble is used to evaluate it. When the spin and valley splittings are added to the broadened Landau level spectrum then the experimental results of K.v. Klitzing (K.v. Klitzing, 1981) are reproduced in detail. No other simple model has been demonstrated to do this. Moreover at higher temperature the model gives the classical results.

Interesting future tasks in connection with this microscopic IQHE model are:

The effect of including higher order interaction effects (for example Coulomb interaction) in the evaluation of n_e has to be studied, especially in connection with "pre-plateau dips" of σ_H seen in experiments (K.v. Klitzing, 1980, 1981).

The dependence of g* on the magnetic field and the number of Landau levels has to be studied in order to get good fits to experiments performed on GaAs.

The number of states density function describing the broadening of the Landau levels (2.265) has to be studied with respect to mini-gaps that could explain the FQHE on the basis of the model presented in this thesis.

In connection with the FQHE it is shown that in the Tao-Thouless

model (R. Tao, 1983) the assumed form for the self consistent Green's functions violates the current conservation. This emphasizes the importance of using exact relations as the W-T relations to ensure that approximations to the response functions or the Green's functions reflect the initial symmetries of the model. The derivation of the excitation gap in the T-T model has to be re-examined, is the gap really caused by the Coulomb interaction? When the gap is assumed instead to represent the effect of the previously neglected higher Landau levels, then the singularity in the plasmon dispersion vanishes. Another important related task in connection with the T-T model is to study the choice of the groundstate by a microscopic calculation. This is important since the model predicts the FQHE to occure both for odd and even denominal filling factors, contrary to the experimental results to date. Another model of the FQHE is the set of groundstate trial wavefunctions suggested by R.B. Laughlin (R.B. Laughlin, 1983). In section 1.5.7 it is shown that the exact Hamiltonian of this model does not include interactions between the electrons, in contrast to claims made by S.M. Girvin et. al. (S.M. Girvin, 1983).

It remains an open question whether these wavefunctions of Laughlin are related to the assumed groundstate in the T-T model (R. Tao, 1983).

In the general context of the FQHE one needs to explore if the transverse modes are of importance there, since it has been noticed (2.297) that the current in the system, when only one Landau level is occupied and all higher ones are neglected, is entirely transverse. The accuracy of the FQHE may also hint that, contrary to the commonly held belief, the fractional effect might not be a many body effect, but rather a reflection of some internal symmetries. In either case the

importance has to be placed upon building a microscopic model based on a Hamiltonian rather than trial wavefunctions.

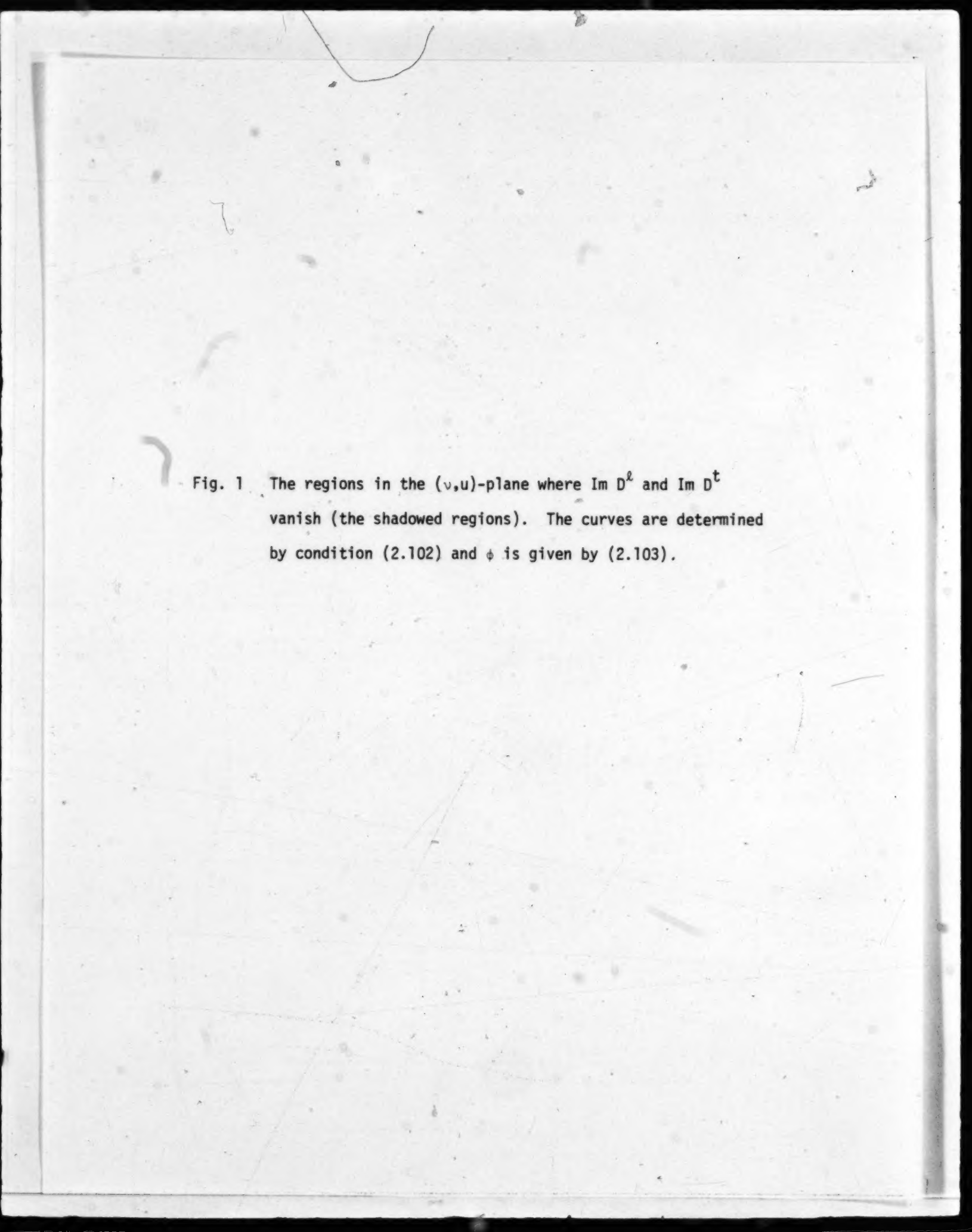


Fig. 1 The regions in the (ν,u)-plane where Im D^{ℓ} and Im D^{t} vanish (the shadowed regions). The curves are determined by condition (2.102) and ϕ is given by (2.103).

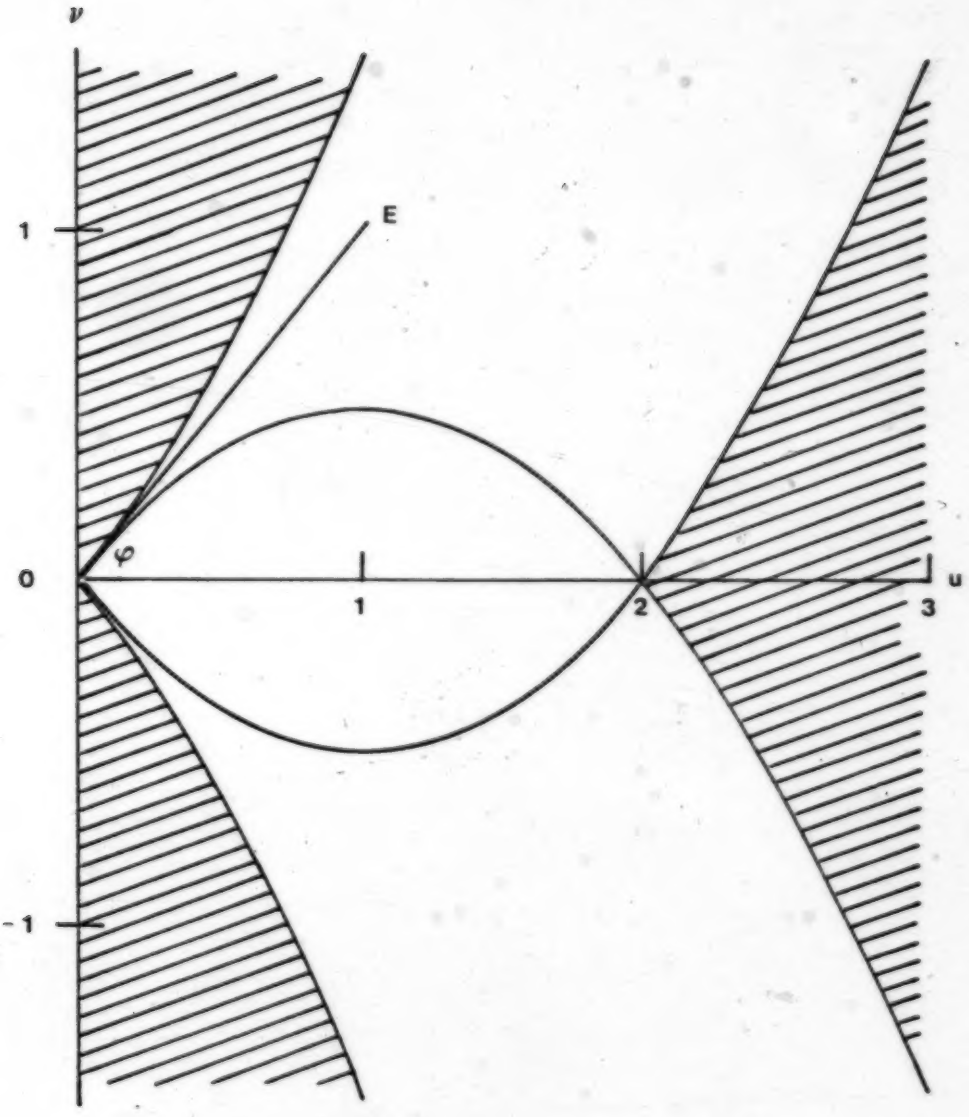
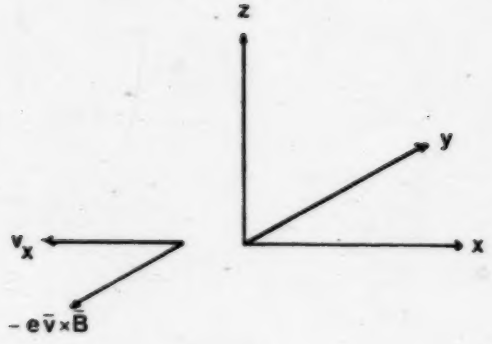

v
1
E
φ
0
1
2
3
u
-1

Fig. 2 The geometry of Hall effect experiments, see eq.'s (2.198) and (2.199). (Based on (N.W. Ashcroft, 1976))


z
y
x
v_x
$-e\bar{v}\times\bar{B}$

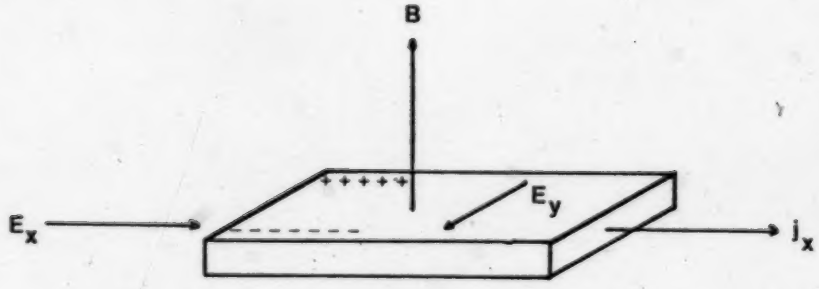

B
E_x
E_y
j_x

Fig. 3 A MOSFET. S, D and G stand for source, drain and gate respectively, while V_g is the gate voltage which regulates the density of the 2D electron gas (2D EG).(Based on (S. Kawaji, 1983))

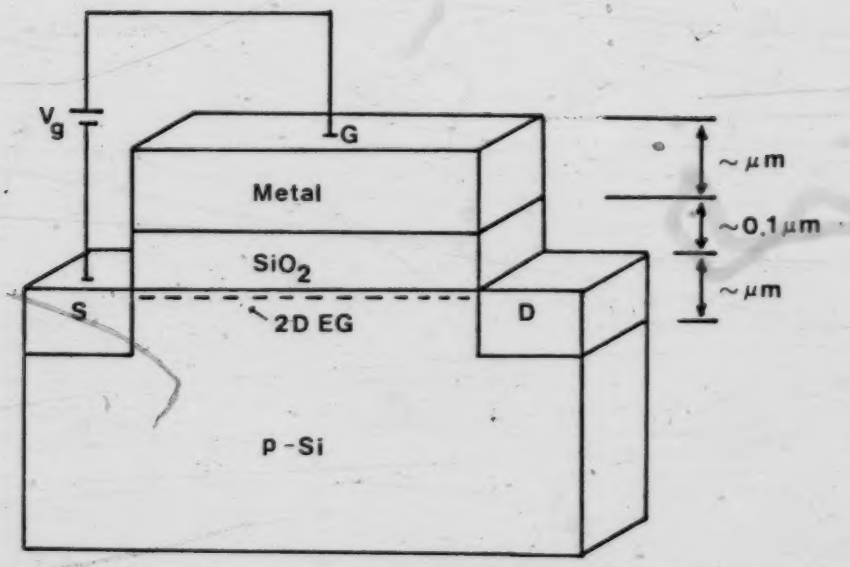

V_g
G
Metal
SiO_2
S
2D EG
D
p-Si
~μm
~0.1μm
~μm

Fig. 4 The energy bands of a p-type Si at the Si-SiO_2 interface in a MOSFET. (a) The flat bands in the case of no gate bias. (b) Depletion of holes near the interface due to a positive gate bias. (a) Band bending due to a strong positive gate bias. An inversion layer of 2D electrons is formed at the interface. E_c and E_v are the conduction and valence bands, E_f is the Fermi level and E_a is the energy of the acceptors. (Based on (T. Ando, 1982a))

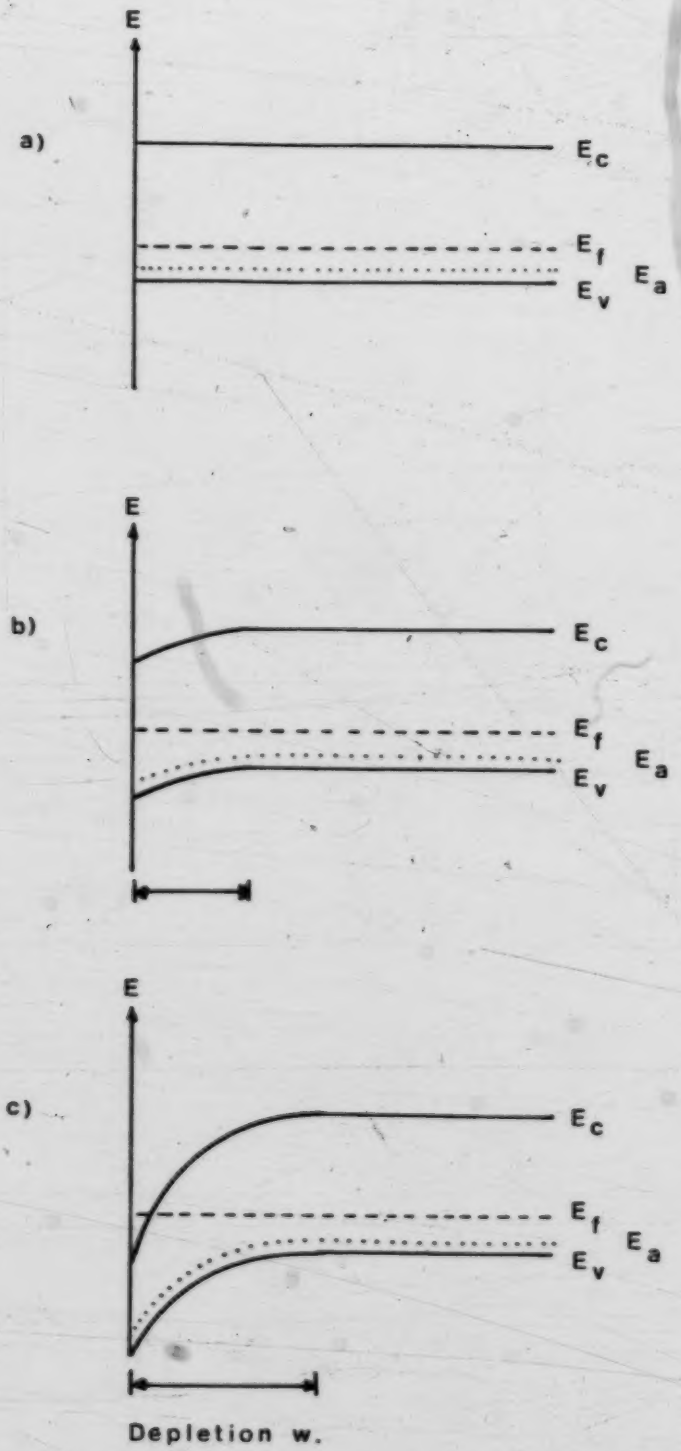

a)
E
E_c
E_f
E_v
E_a
b)
E
E_c
E_f
E_v
E_a
c)
E
E_c
E_f
E_v
E_a
Depletion w.

Fig. 5 A GaAs heterostructure, the 2D electron gas (2D EG) resides in the GaAs layer near the $AL_xGa_{1-x}As$ - GaAs interface. (Based on (S. Kawaji, 1983))

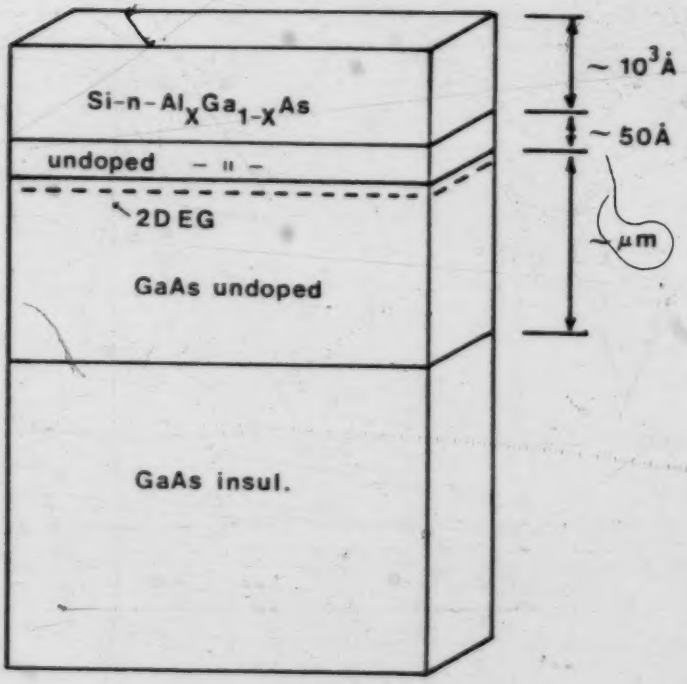

Si-n-$Al_xGa_{1-x}As$
undoped - " -
2D EG
GaAs undoped
GaAs insul.
~ 10^3 Å
~ 50 Å
~ µm

Fig. 6 The bandstructure of a GaAs heterostructure. The 2D electron gas (2D EG) resides in the discontinuity of the conduction band E_c. E_v and E_f are the energies of the valence band and the Fermi level. (Based on (H.L. Störmer, 1984))

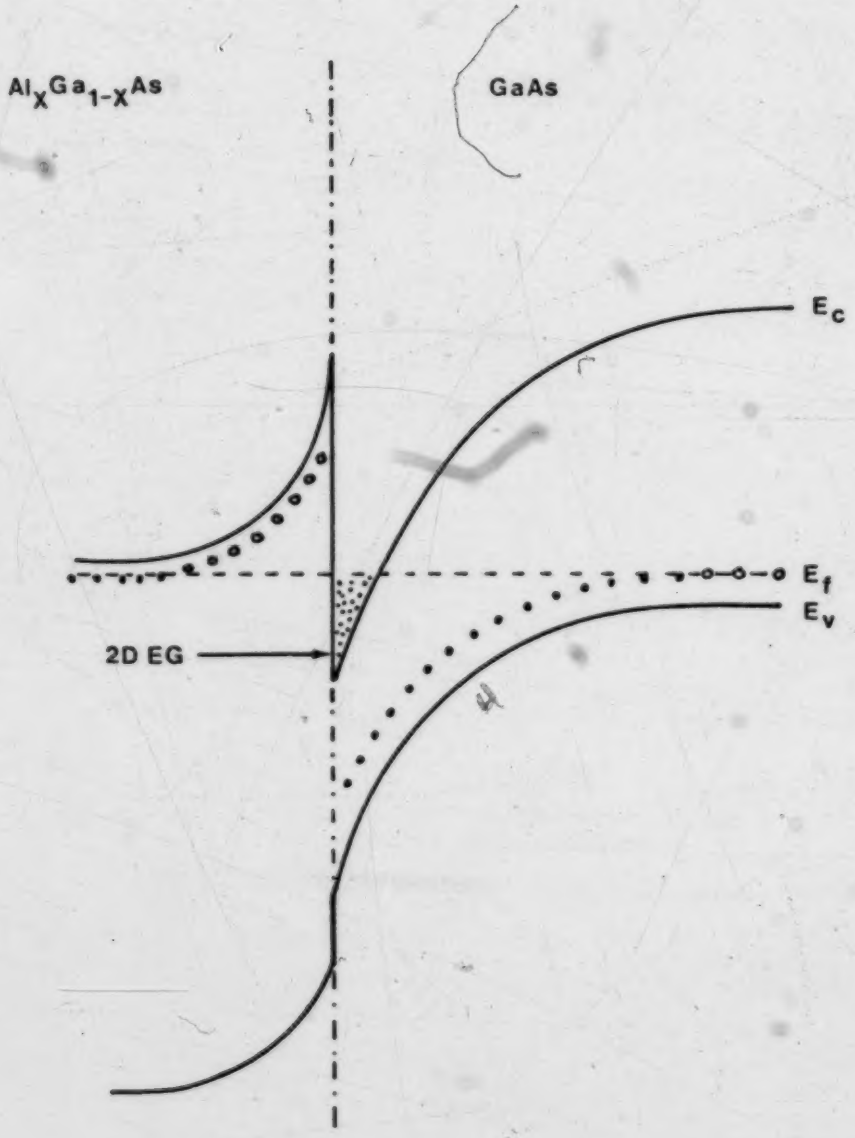

$Al_xGa_{1-x}As$
GaAs
E_c
E_f
E_v
2D EG

Fig. 7 The experimental results of K.v. Klitzing et. al. (K.v. Klitzing, 1981) for the IQHE in a MOSFET. The resistivities ρ_{xy} and ρ_{yy} are shown as functions of the gate voltage V_g. The magnetic field is held at B = 18.9T and the temperature is 1.5 K. The extent of the Landau levels n=1, and n=2 with respect to V_g is shown.

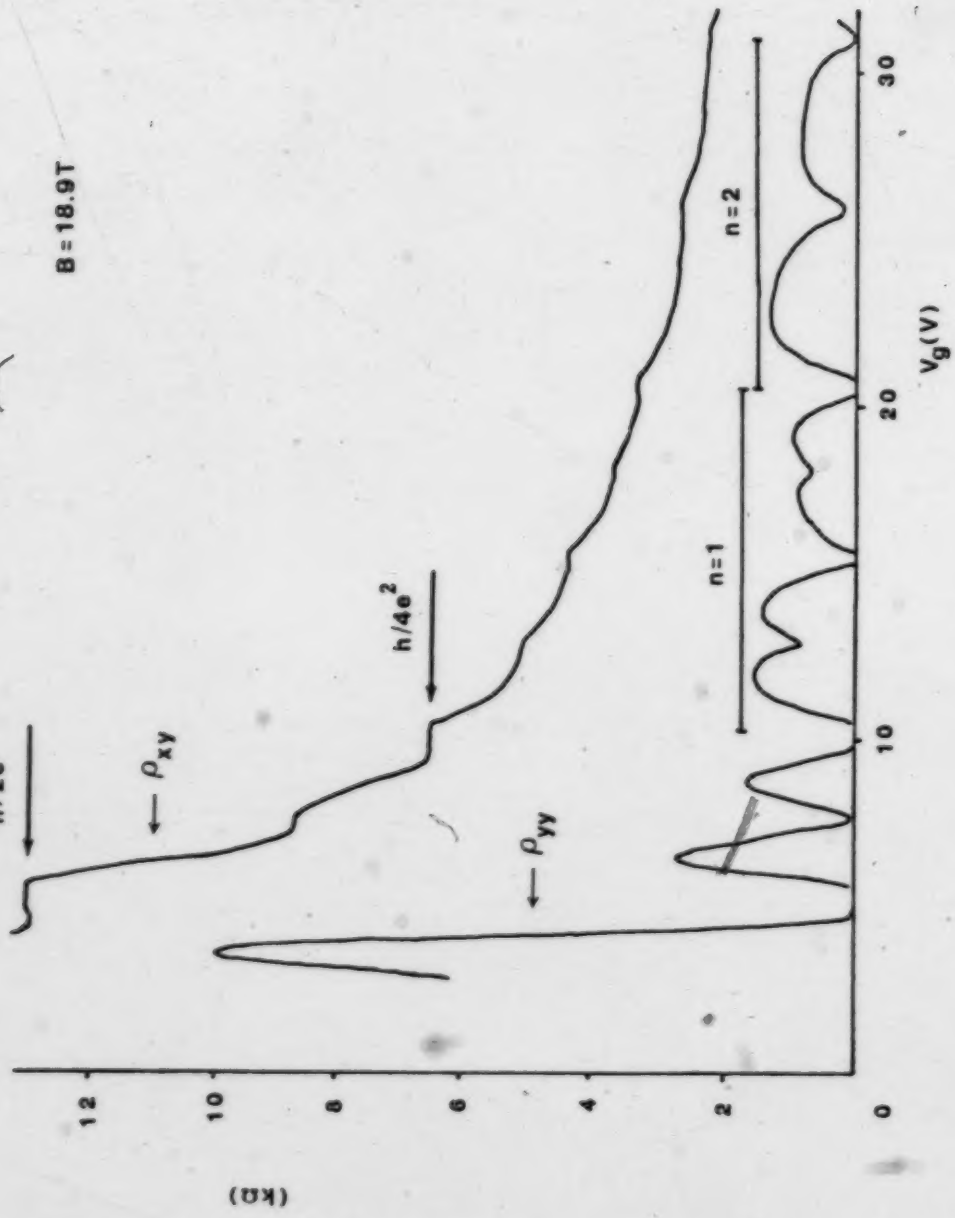

B=18.9T
h/2e²
ρxy
h/4e²
ρyy
n=1
n=2
(kΩ)
12
10
8
6
4
2
0
10
20
30
Vg(V)

Fig. 8 The resistivities ρ_{xx} and ρ_{xy} as functions of the magnetic field B, in an experiment on a GaAs-heterostructure at T = 50 mK (M.A. Paalanen, 1982). The number of the respective Landau levels is shown on the ρ_{xx} graph together with the spin polarization which is resolved for the n=1 and n=2 levels.

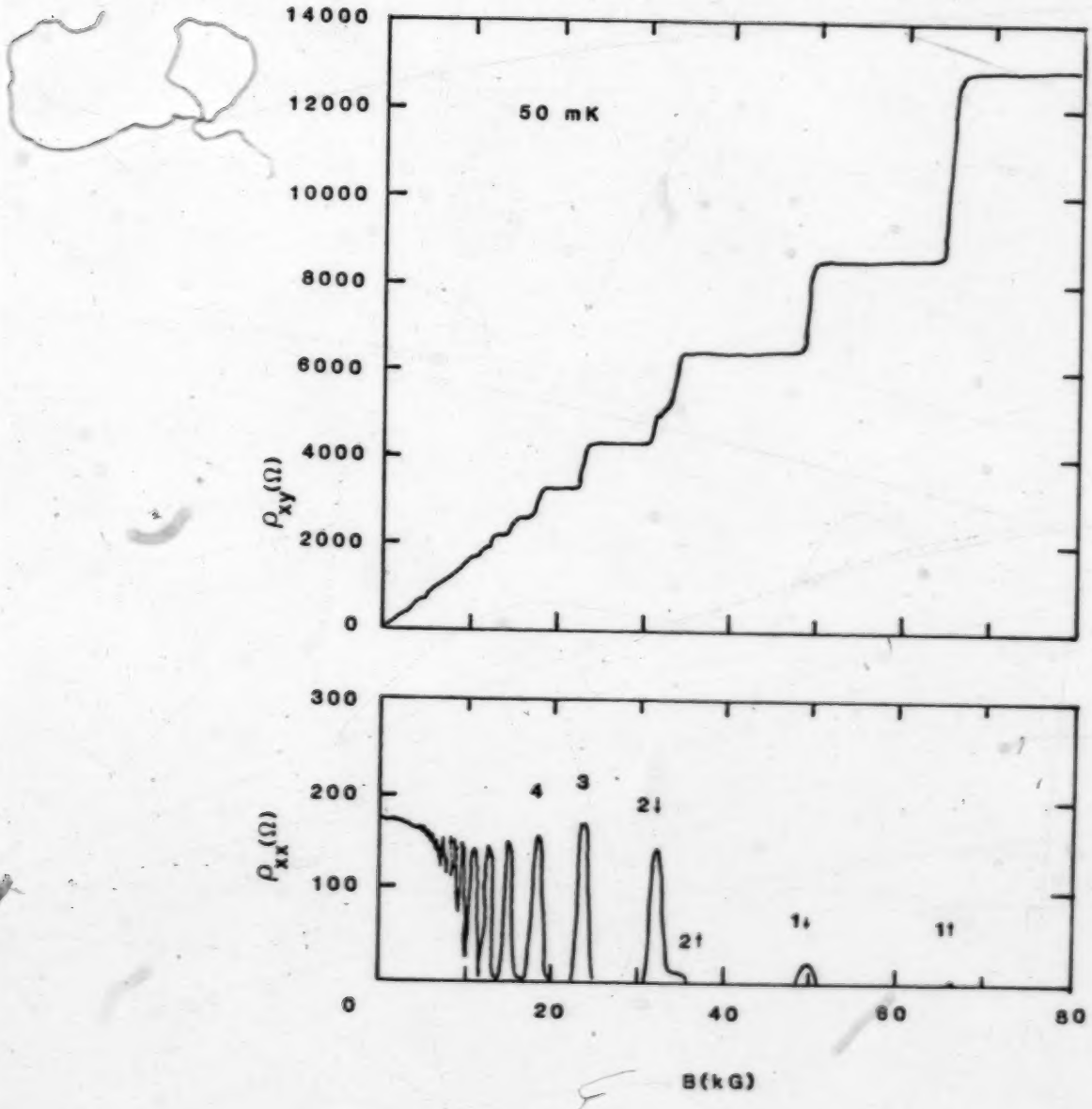

14000
12000
10000
8000
6000
4000
2000
0
50 mK
$\rho_{xy}(\Omega)$
300
200
100
0
$\rho_{xx}(\Omega)$
4
3
2↓
2↑
1↓
1↑
20
40
60
80
B(kG)

Fig. 9 The resistivities ρ_{xx} and ρ_{xy} as functions of the magnetic field B, in an experiment on a GaAs-heterostructure at T = 8mK (K.v. Klitzing, 1982). The number and spin polarization of the respective Landau levels is shown.

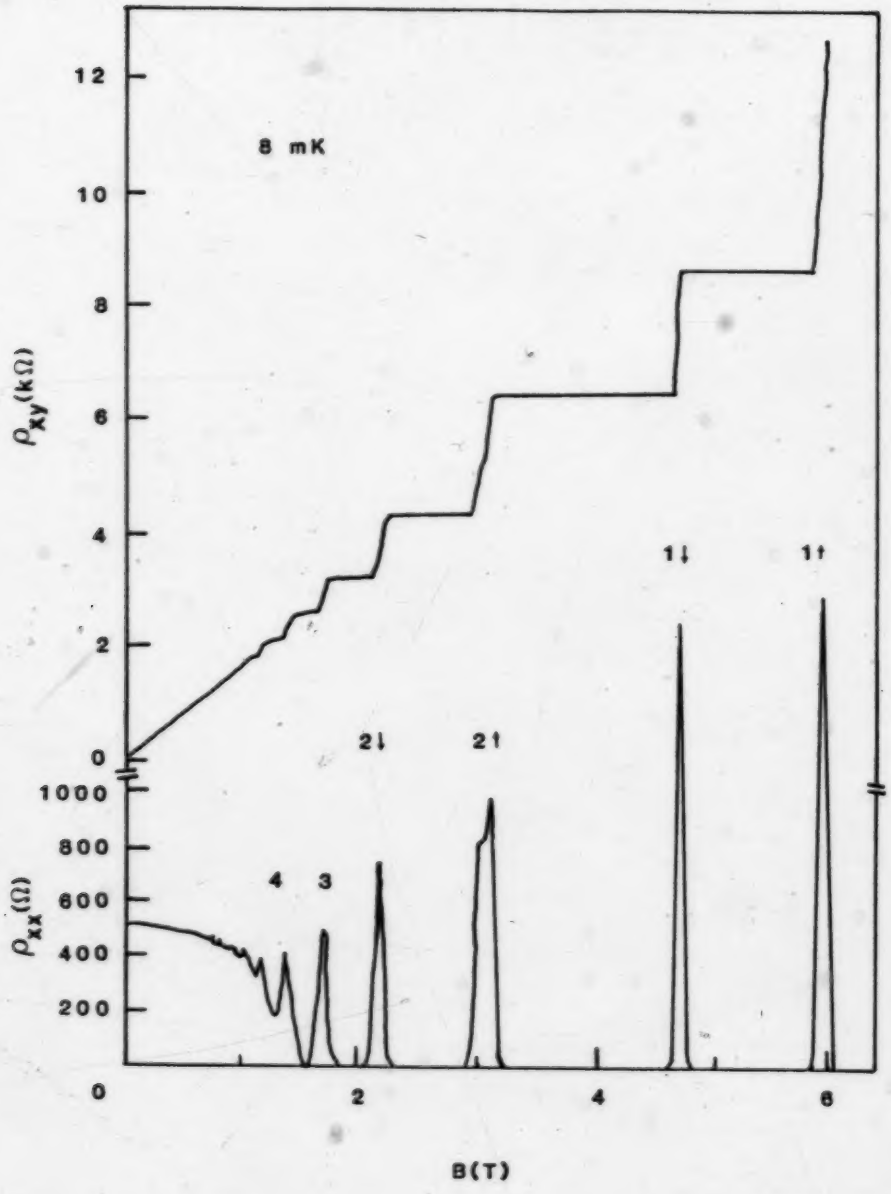

8 mK
ρ_{xy}(kΩ)
ρ_{xx}(Ω)
1↓
1↑
2↓
2↑
4
3
B(T)

Fig. 10 The FQHE in GaAs heterostructure, the resistivities ρ_{xy} and ρ_{xx} are shown as functions of the magnetic field B or the filling factor ν, at T = 0.55K.(H.L. Störmer, 1983a)

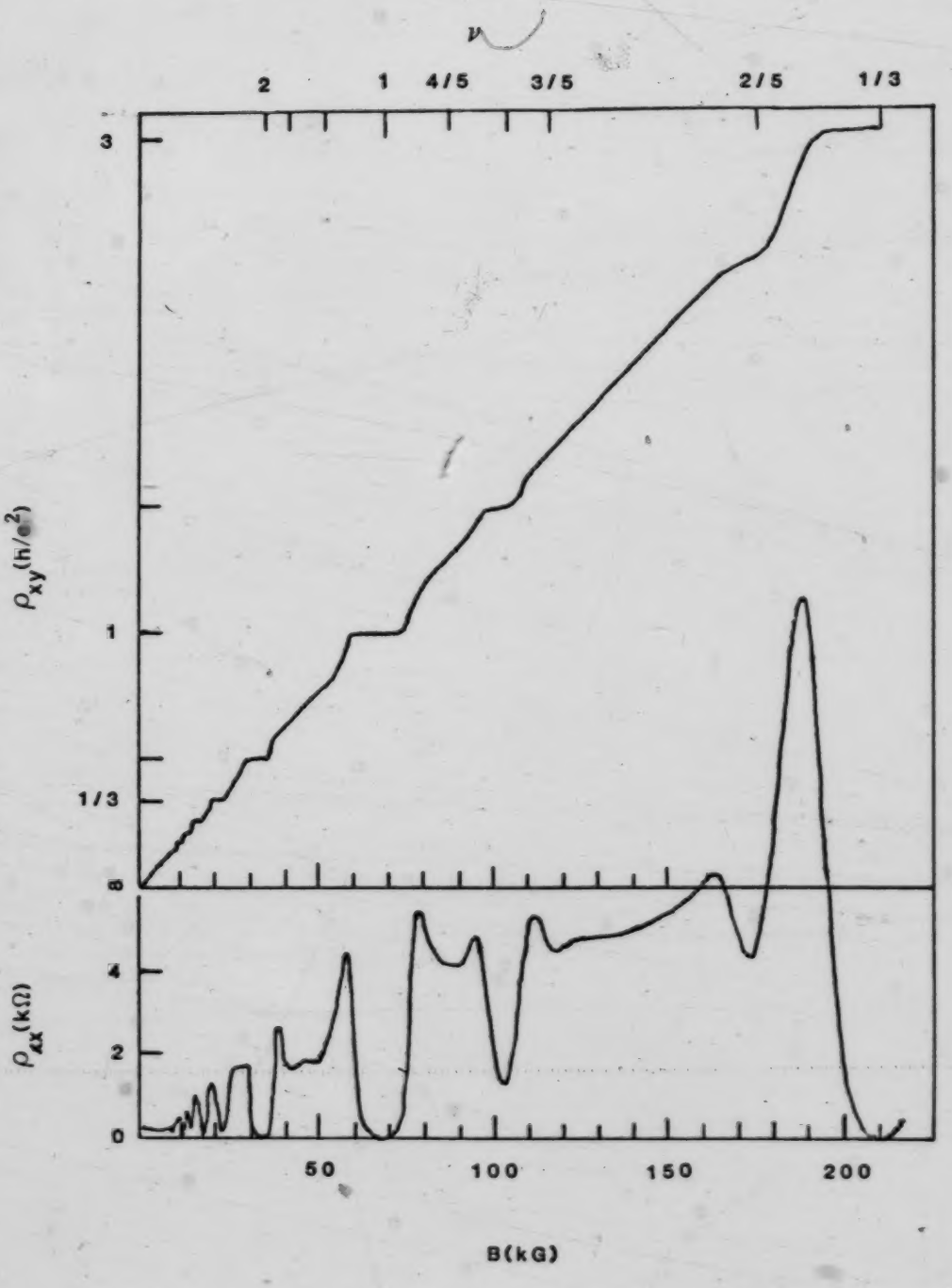

ν
2
1
4/5
3/5
2/5
1/3
3
1
1/3
0
ρ_{xy} (h/e^2)
4
2
0
ρ_{xx} (kΩ)
50
100
150
200
B(kG)

Fig. 11 The FQHE in a GaAs heterostructure, the resistivities ρ_{xy} and ρ_{xx} are shown as functions of the magnetic field B or the filling factor ν, at T = 0.55K (K.v. Klitzing, 1984).

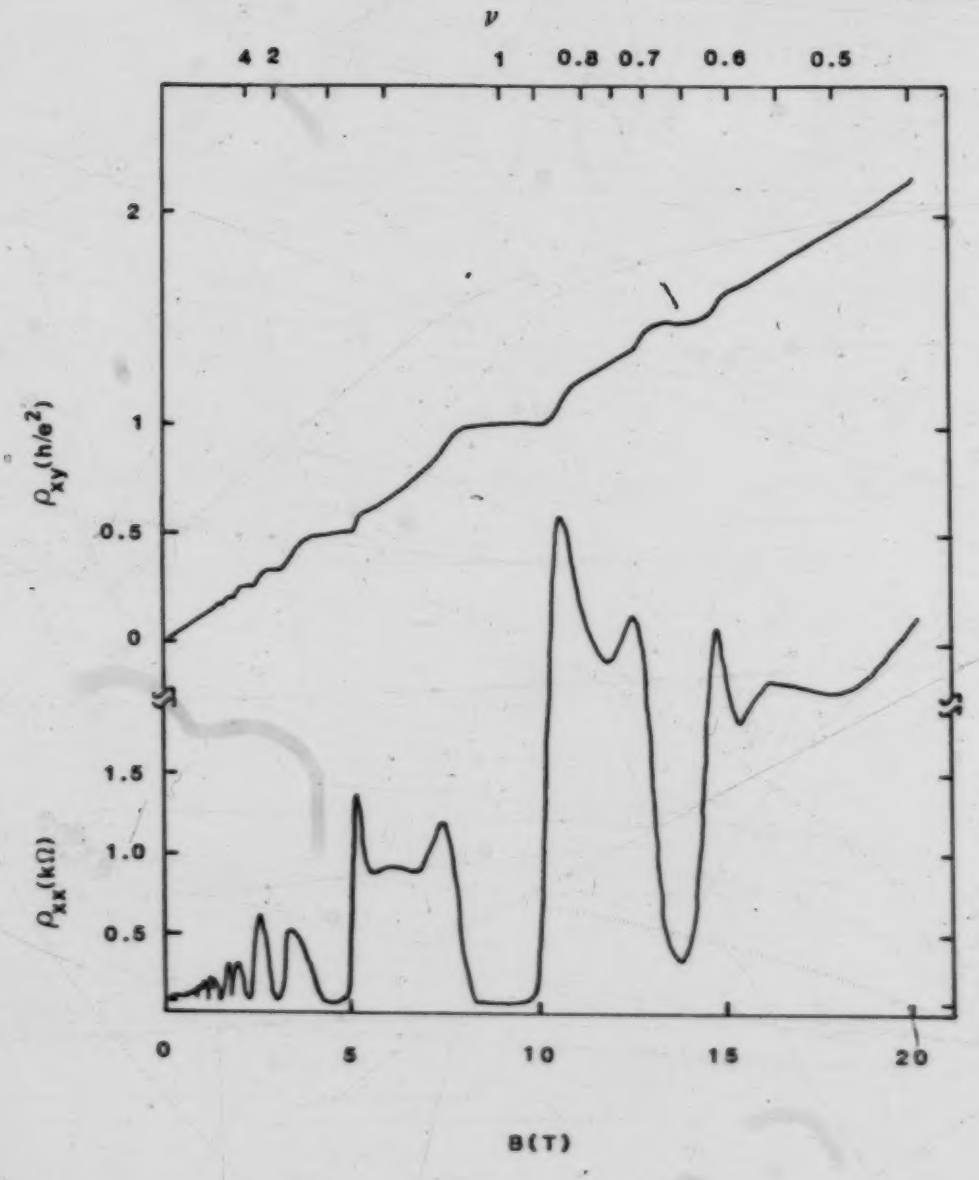

ν
4 2
1
0.8 0.7
0.6
0.5
2
1
0.5
0
$\rho_{xy}(h/e^2)$
1.5
1.0
0.5
$\rho_{xx}(k\Omega)$
0
5
10
15
20
B(T)

Fig. 12 The temperature dependence of the FQHE. The resistivities ρ_{xy} and ρ_{xx} are shown as functions of the magnetic field B or the filling factor ν, at various temperatures.(D.C. Tsui, 1982)

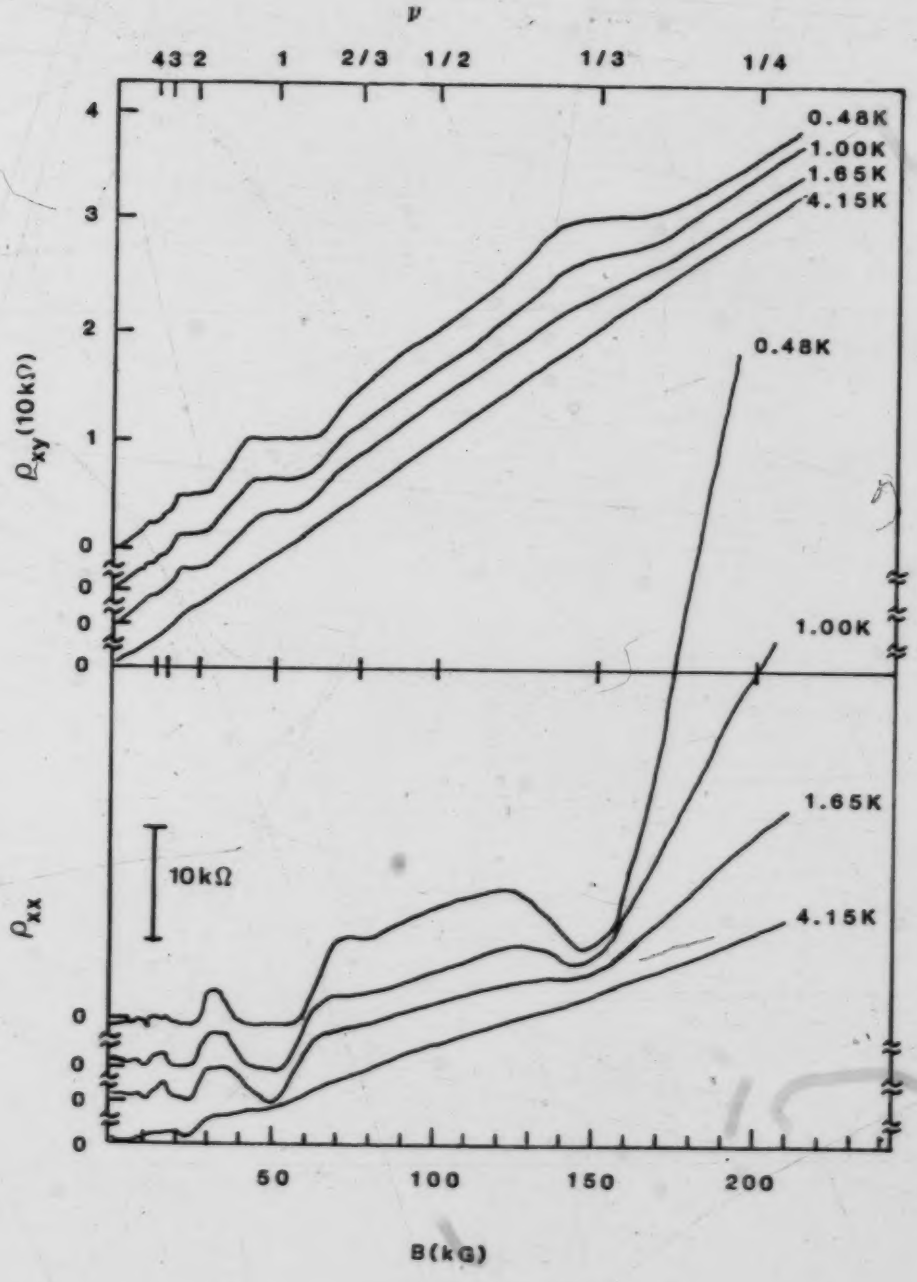

ν
4 3 2
1
2/3
1/2
1/3
1/4
0.48K
1.00K
1.65K
4.15K
ρ_{xy} (10kΩ)
0.48K
1.00K
1.65K
4.15K
10kΩ
ρ_{xx}
50
100
150
200
B(kG)

Fig. 13 The quantum Hall conductivity σ_H, from eq. (2.270), as a function of the chemical potential μ. The smoother curve represents the temperature T = 5.8K, and the sharper one T = 0.58K. The Landau level line shape is assumed Gaussian eq. (3.4) with half width Γ = 0.5 meV, and $\hbar\omega_c$ = 10 meV.

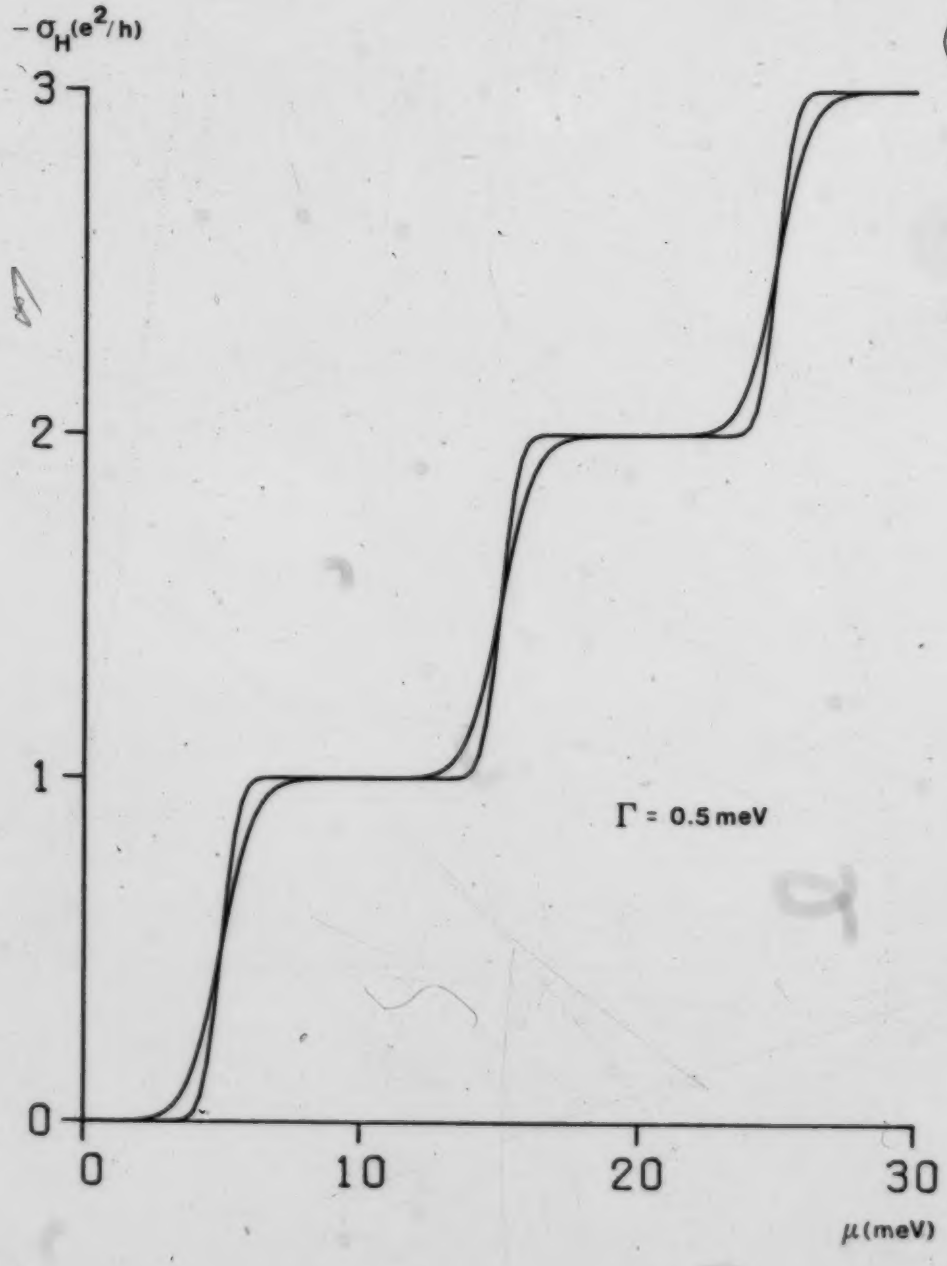

$-\sigma_H(e^2/h)$
3
2
1
0
0
10
20
30
μ(meV)
Γ = 0.5 meV

Fig. 14 The quantum Hall conductivity σ_H, from eq. (2.270), as a function of the chemical potential μ. The smoother curve represents the temperature T = 5.8K, and the sharper one T = 0.58K. The Landau level line shape is assumed Gaussian eq. (3.4) with half width Γ = 1.0 meV, and $\hbar\omega_c$ = 10 meV.

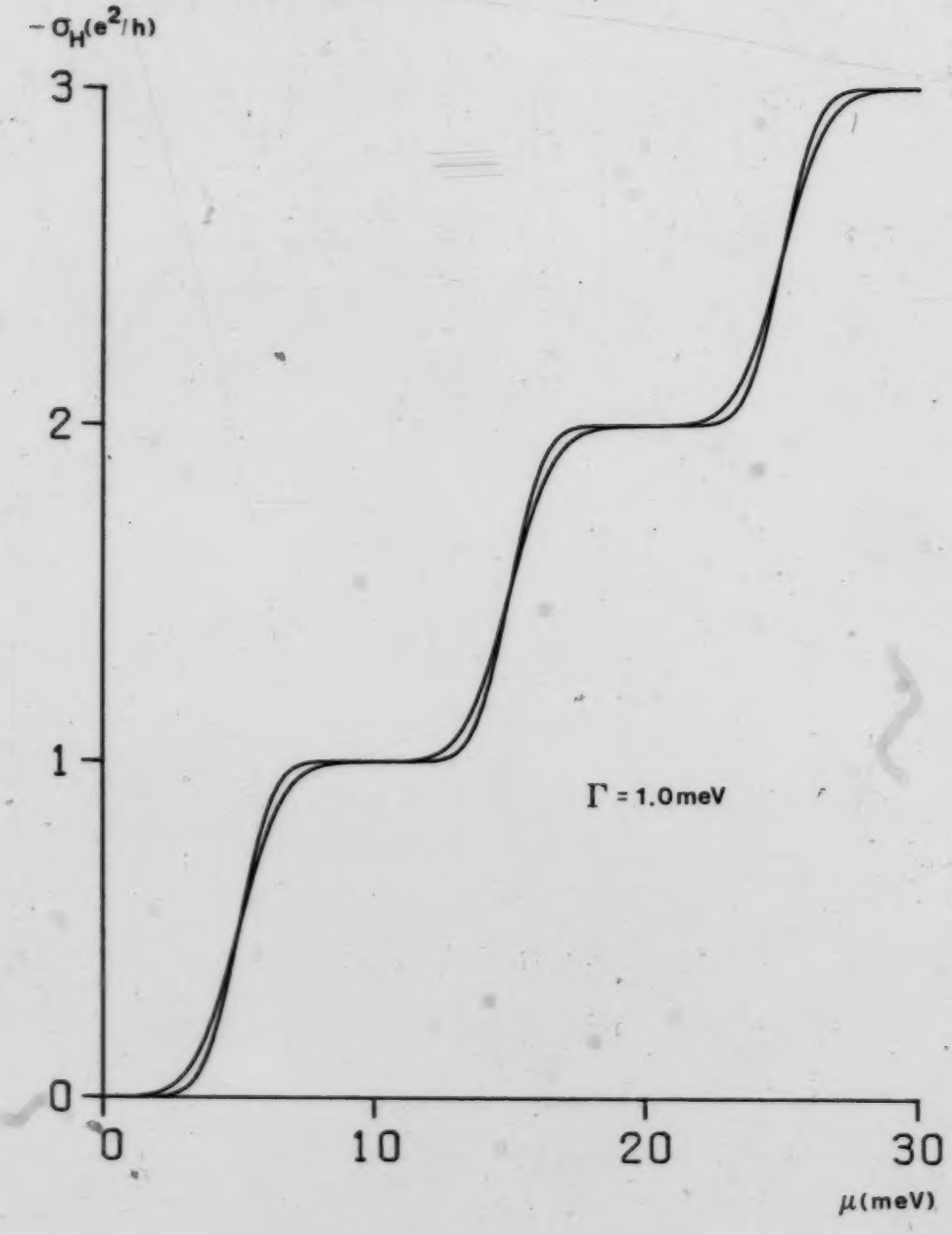

$-\sigma_H(e^2/h)$
3
2
1
0
0
10
20
30
μ(meV)
Γ = 1.0meV

Fig. 15 The quantum Hall conductivity σ_H, from eq. (2.270), as a function of the chemical potential μ. The smoother curve represents the temperature T = 5.8K, and the sharper one T = 0.58K. The Landau level line shape is assumed Gaussian eq. (3.4) with half width Γ = 2.0 meV, and $\hbar\omega_c$ = 10 meV.

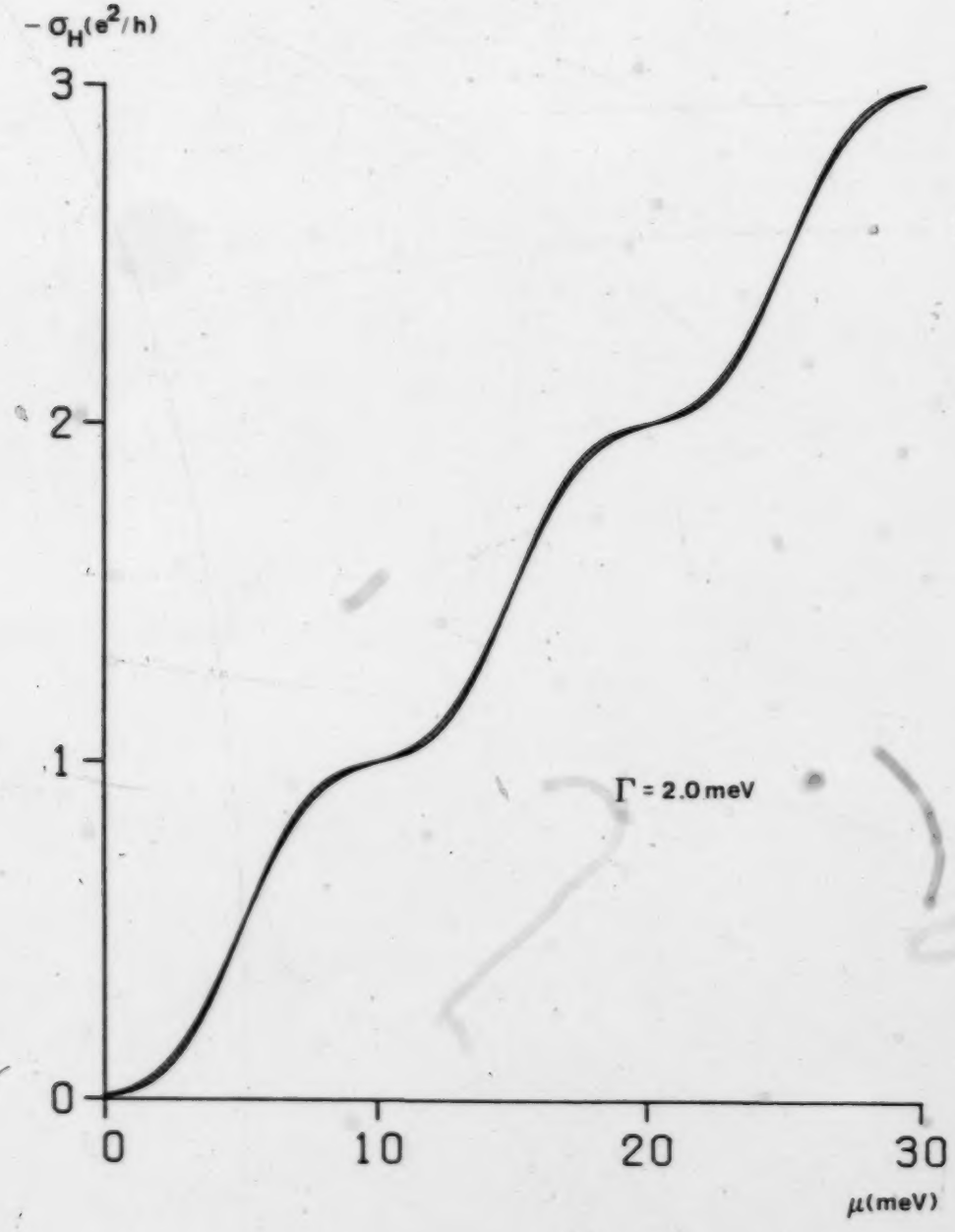

$-\sigma_H(e^2/h)$
3
2
1
0
0
10
20
30
μ(meV)
Γ = 2.0 meV

Fig. 16 The inverse of the absolute value of the Hall conductivity σ_H as a function of the gate voltage V_g at B = 18.9T and T = 1.5K. The solid curve is from eq. (3.6) and (3.7) with the parameter choice (3.8). The dotted curve is from the experimental results of K.v. Klitzing et. al. (K.v. Klitzing, 1981)

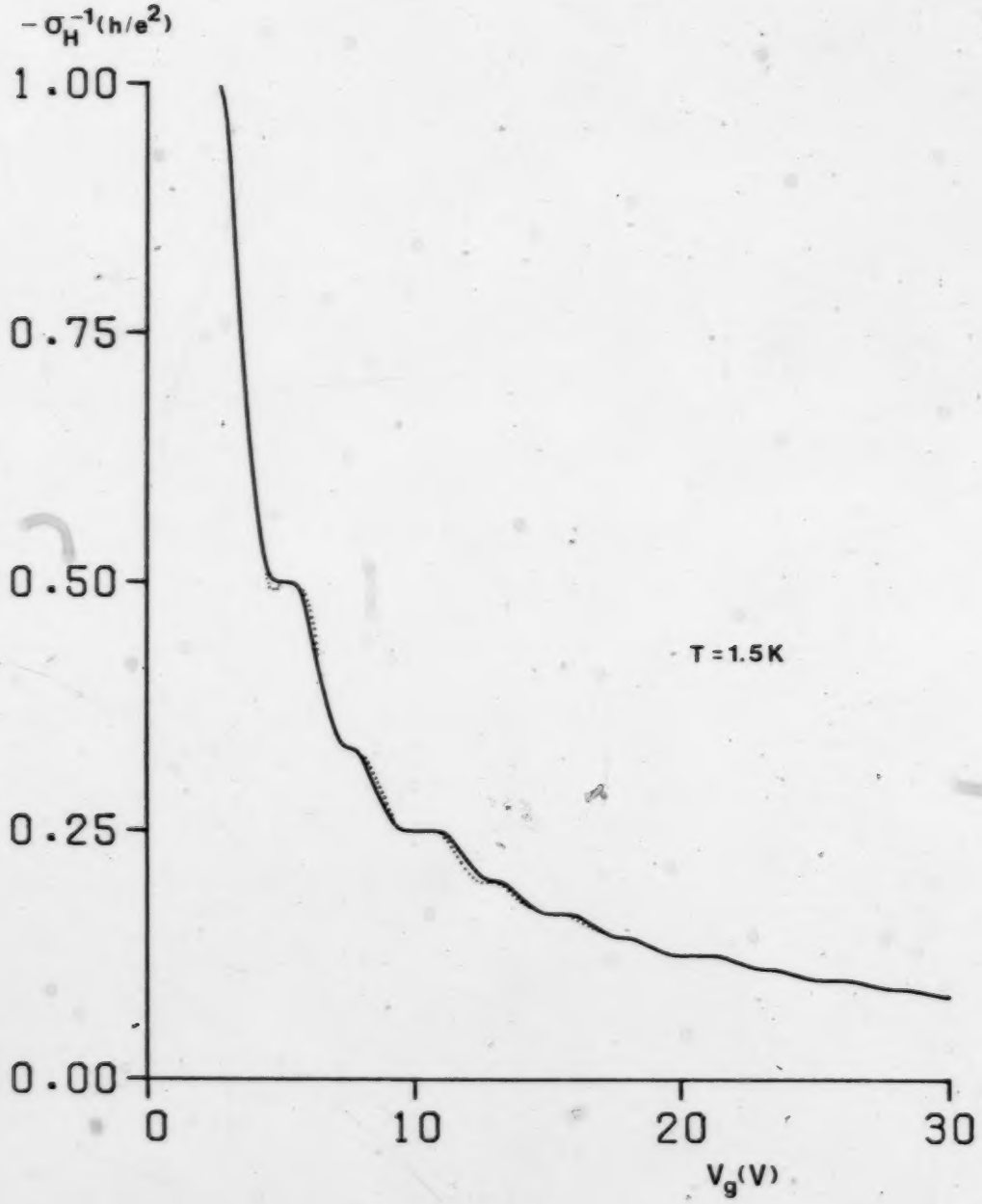

$-\sigma_H^{-1}(h/e^2)$
1.00
0.75
0.50
0.25
0.00
T = 1.5K
0
10
20
30
V_g(V)

Fig. 17 The inverse of the absolute value of the Hall conductivity σ_H as a function of the gate voltage V_g at B = 18.9T and T = 0.5K. The solid curve is from eq. (3.6) and (3.7) with the parameter choice (3.8).

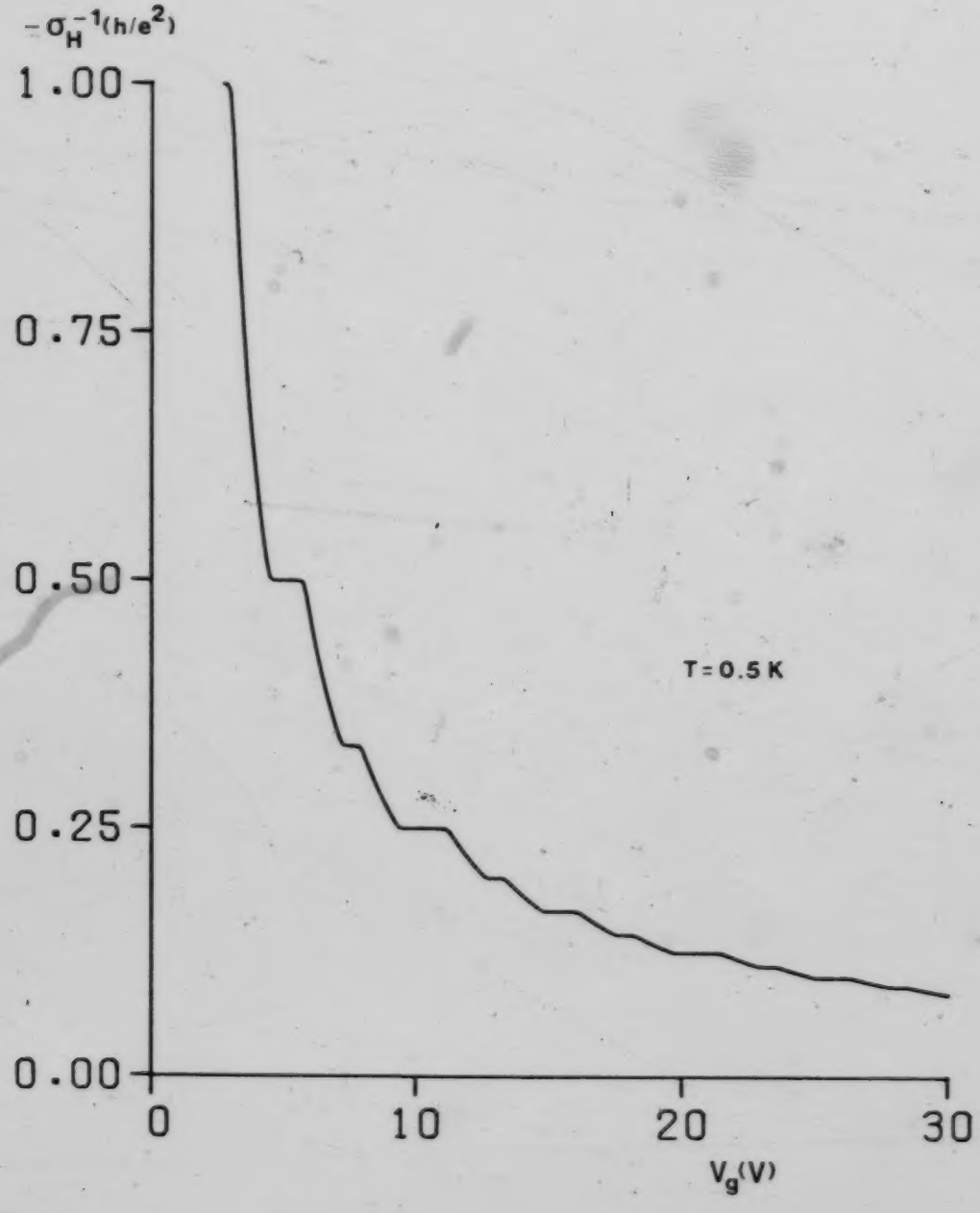

$-\sigma_H^{-1}(h/e^2)$
1.00
0.75
0.50
0.25
0.00
T=0.5 K
0
10
20
30
V_g(V)

Fig. 18 The inverse of the absoluate value of the Hall conductivity σ_H as a function of the gate voltage V_g at B = 18.9T and T = 2.0K. The solid curve is from eq. (3.6) and (3.7) with the parameter choice (3.8).

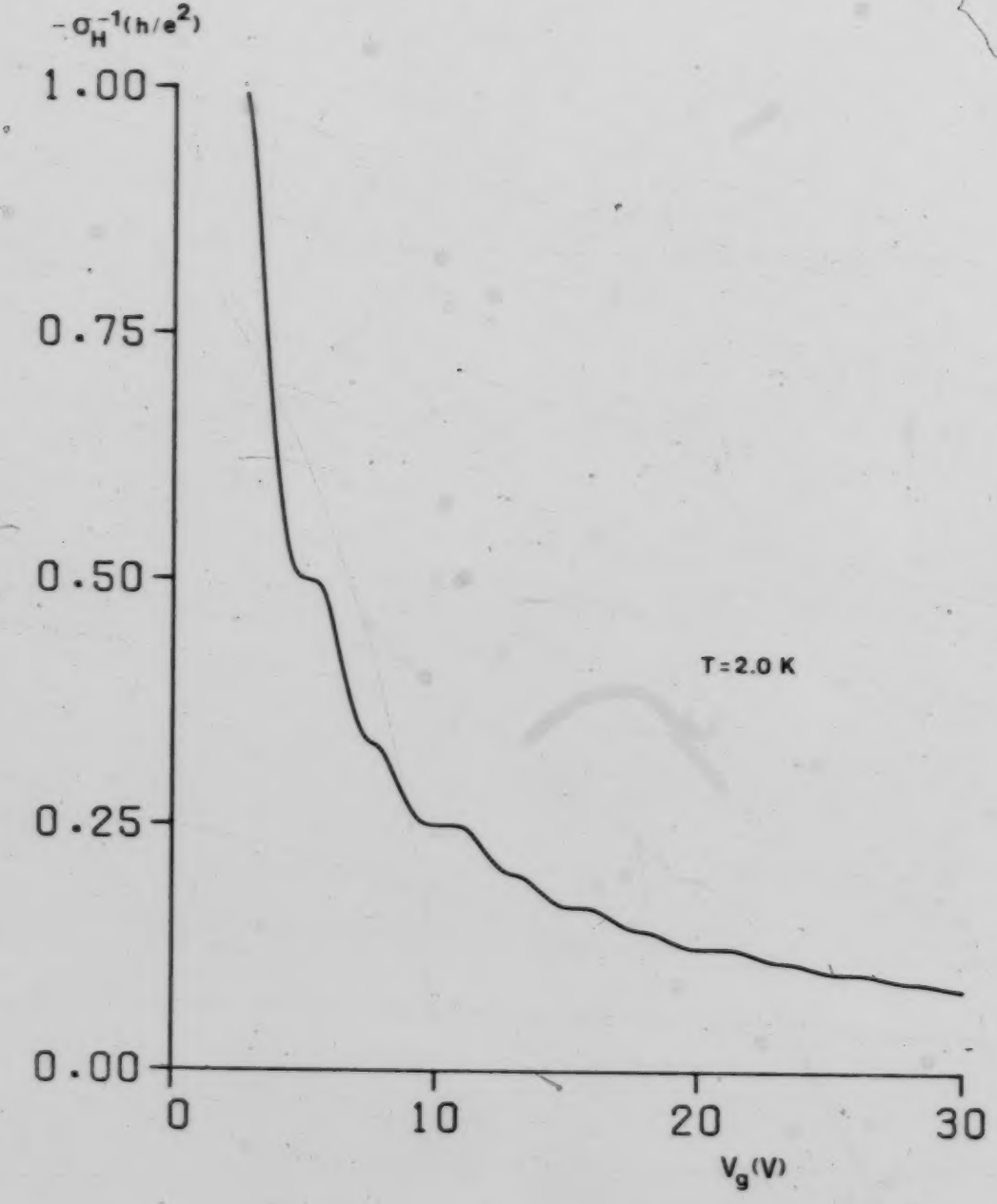

$-\sigma_H^{-1}(h/e^2)$
1.00
0.75
0.50
0.25
0.00
T=2.0 K
0
10
20
30
V_g(V)

Fig. 19 The inverse of the absolute value of the Hall conductivity σ_H as a function of the gate voltage V_g at B = 18.9T and T = 3.0K. The solid curve is from eq. (3.6) and (3.7) with the parameter choice (3.8).

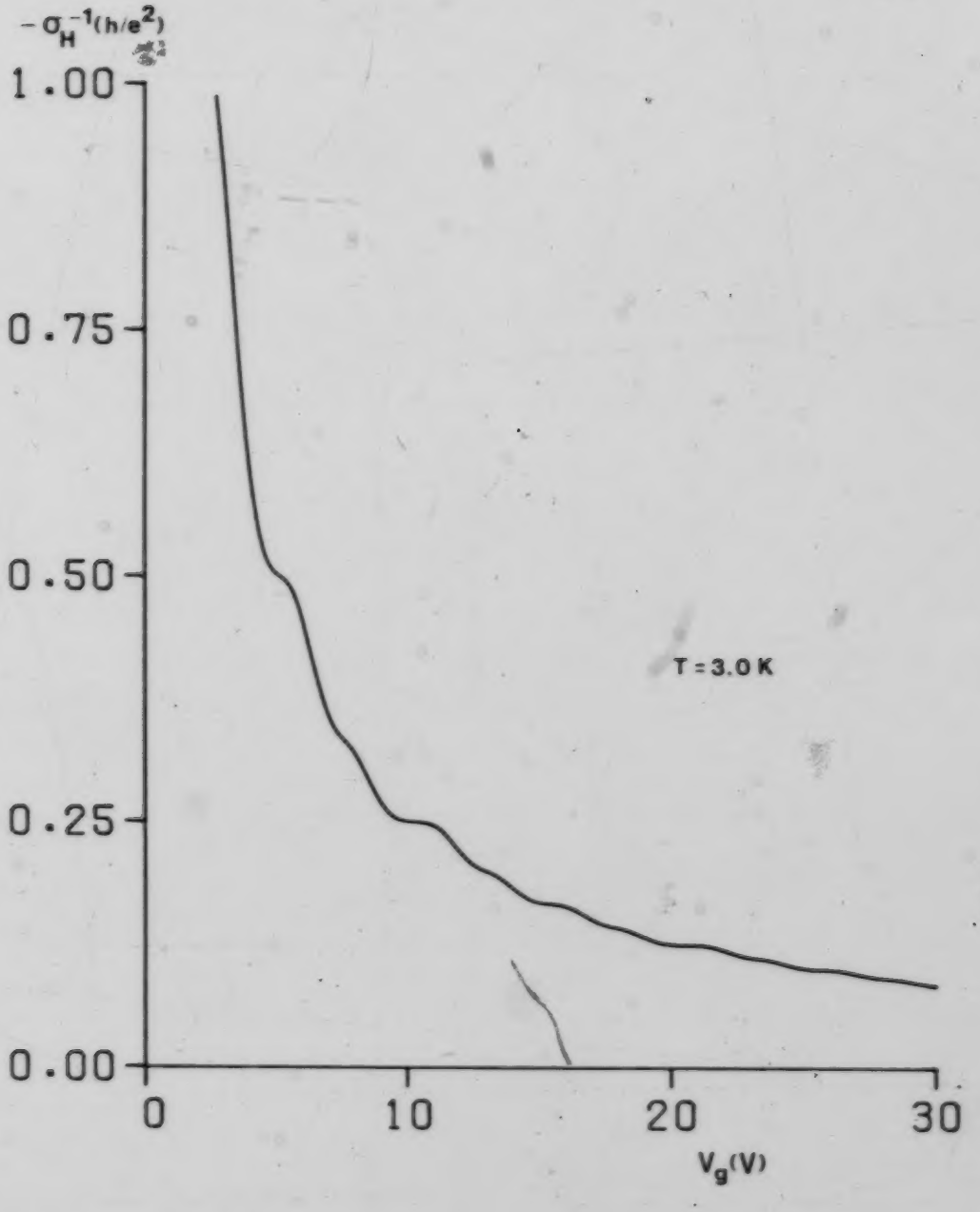

$-\sigma_H^{-1}(h/e^2)$
1.00
0.75
0.50
0.25
0.00
T=3.0 K
0
10
20
30
V_g(V)

Fig. 20 The inverse of the absolute value of the Hall conductivity σ_H as a function of the gate voltage V_g at B = 18.9T and T = 4.2K. The solid curve is from eq. (3.6) and (3.7) with the parameter choice (3.8).

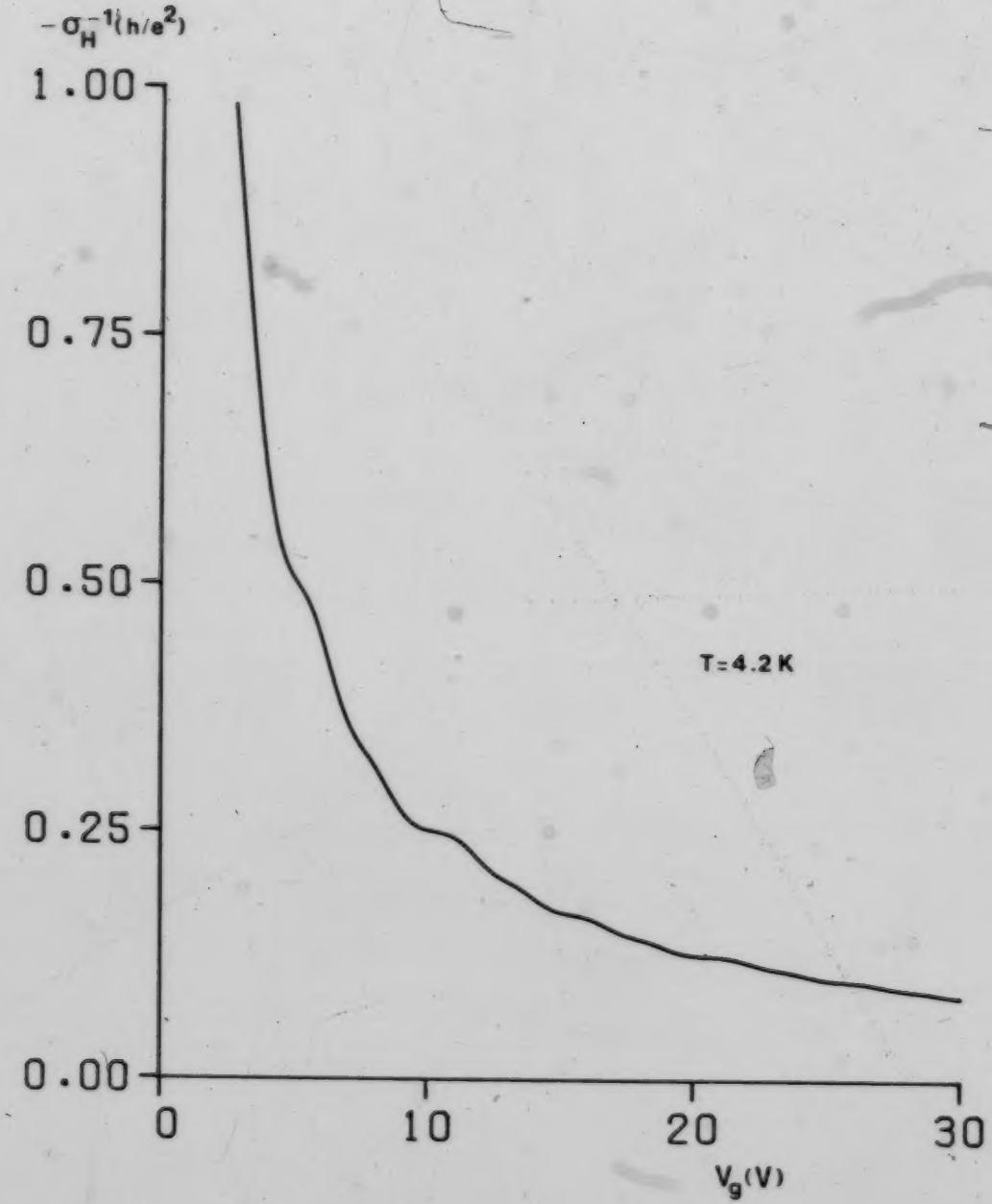

$-\sigma_H^{-1}(h/e^2)$
1.00
0.75
0.50
0.25
0.00
T=4.2 K
0
10
20
30
V_g(V)

Fig. 21 The inverse of the absolute value of the Hall conductivity σ_H as a function of the gate voltage V_g at B = 18.9T and T = 6.0K. The solid curve is from eq. (3.6) and (3.7) with the parameter choice (3.8).

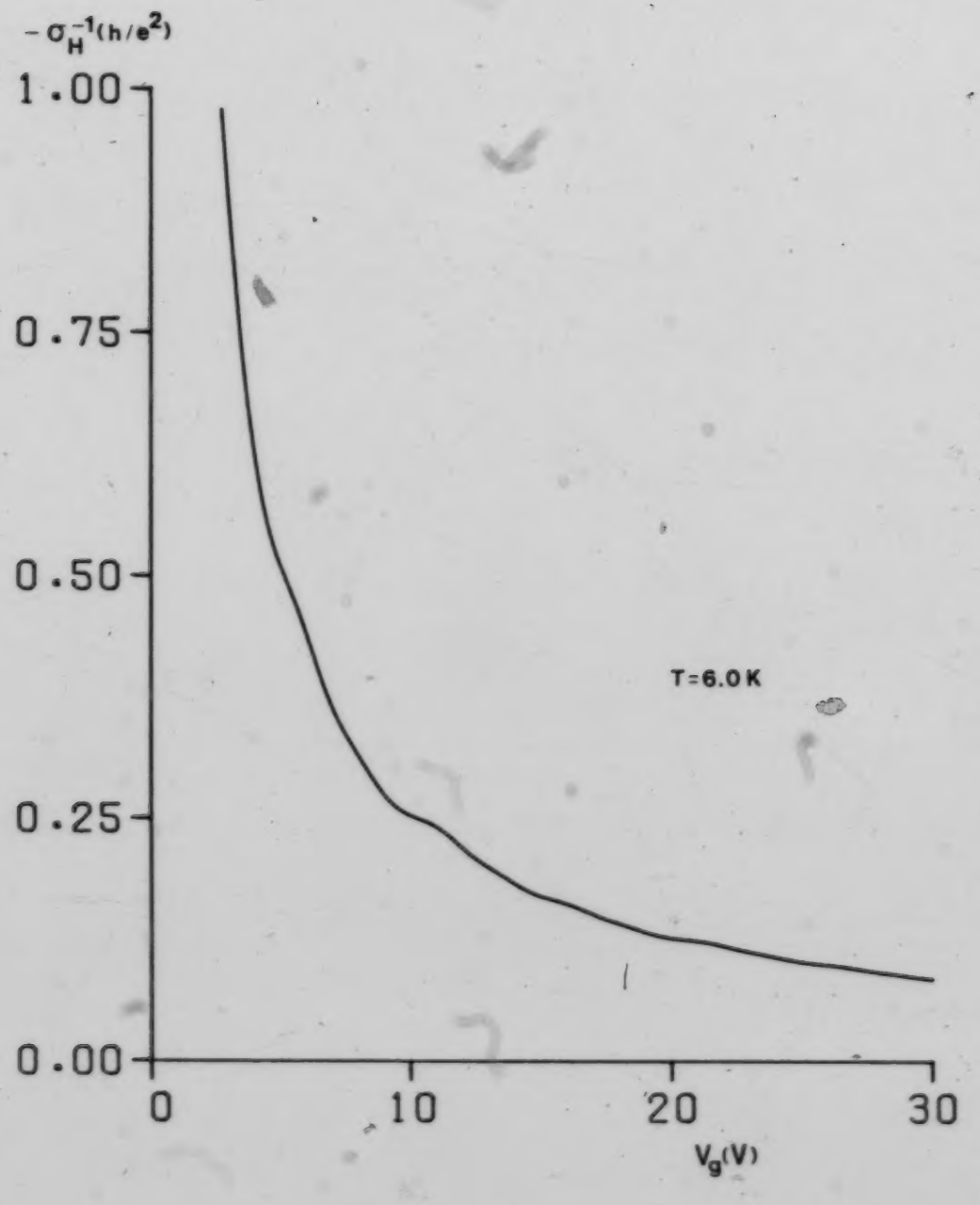

$-\sigma_H^{-1}(h/e^2)$
1.00
0.75
0.50
0.25
0.00
T=6.0K
0
10
20
30
$V_g(V)$

Fig. 22 The inverse of the absoluate value of the Hall conductivity σ_H as a function of the gate voltage V_g at B = 18.9T and T = 10.0K. The solid curve is from eq. (3.6) and (3.7) with the parameter choice (3.8).

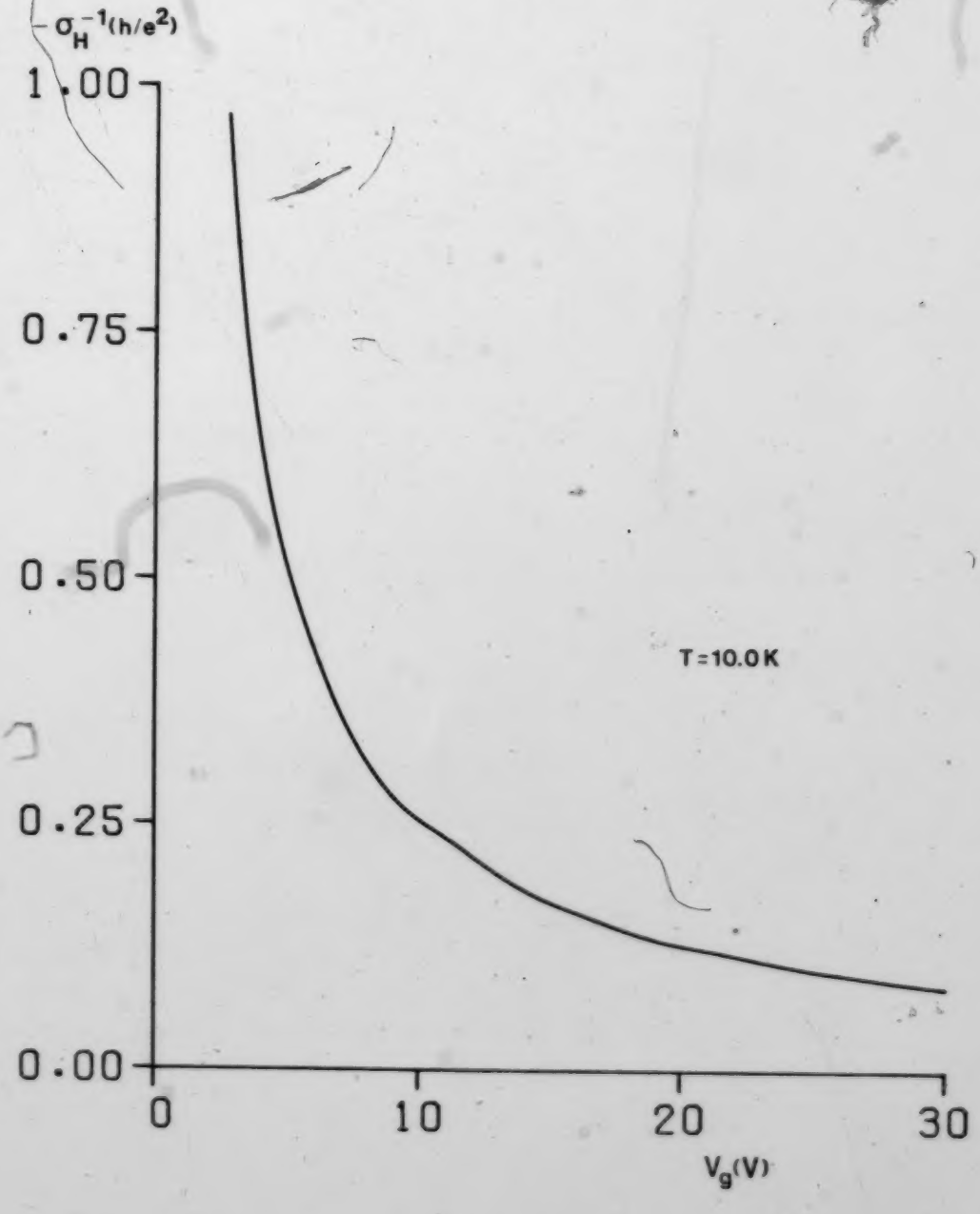
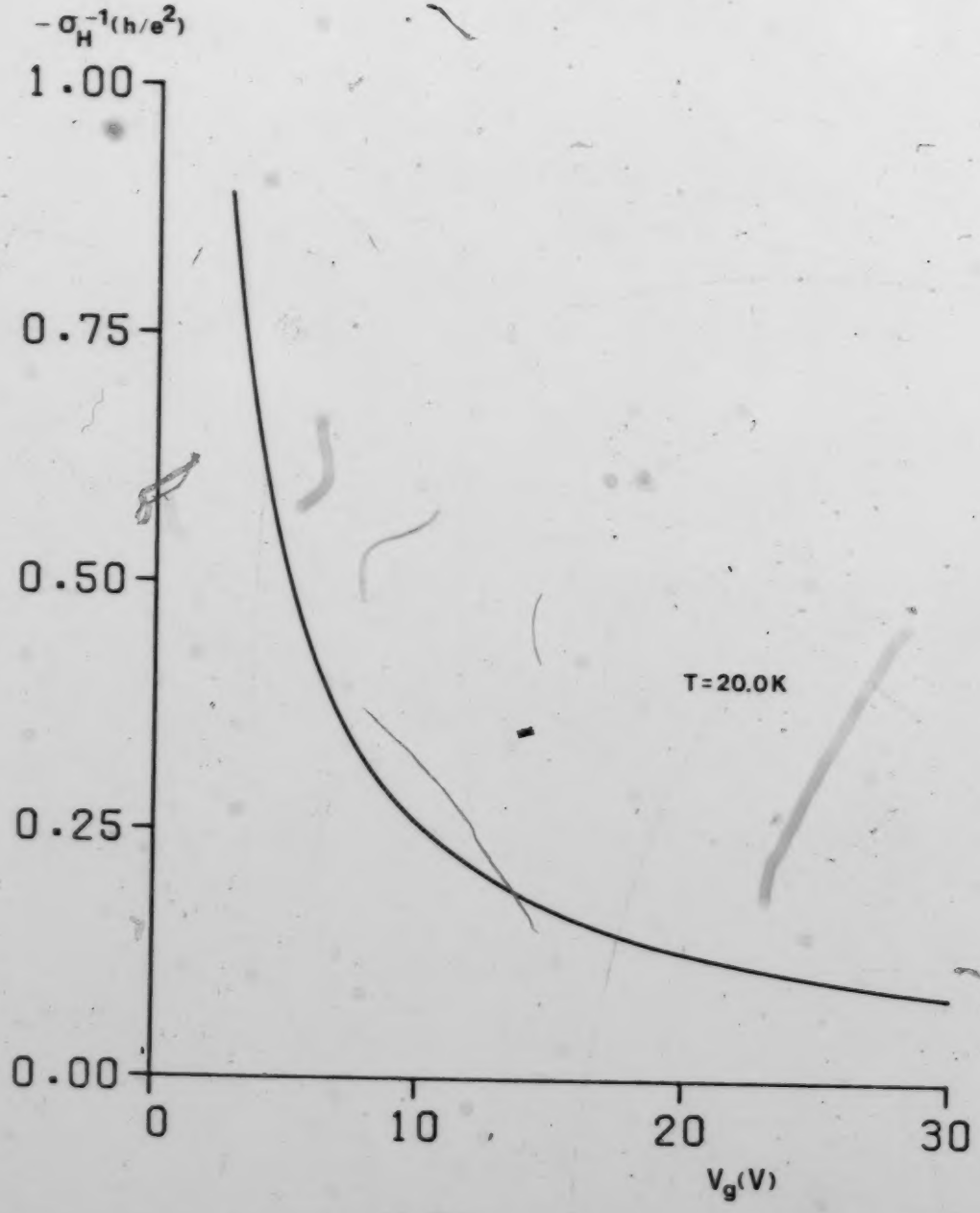

$\sigma_H^{-1}(h/e^2)$
1.00
0.75
0.50
0.25
0.00
T=10.0K
0
10
20
30
$V_g(V)$

Fig. 23 The inverse of the absolute value of the Hall conductivity σ_H as a function of the gate voltage V_g at B = 18.9T and T = 20.0K. The solid curve is from eq. (3.6) and (3.7) with the parameter choice (3.8).


$-\sigma_H^{-1}(h/e^2)$
1.00
0.75
0.50
0.25
0.00
T=20.0K
0
10
20
30
V_g(V)

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   J. Phys. Soc. Jpn. 37 (1974d) 1233.

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APPENDIX A

THE 3D TRANSVERSE PLASMON, DERIVATION ACCORDING TO THE SELF-CONSISTENT LINEAR RESPONSE METHOD

In the 3D case, the linear response of the transverse current is given by:

$$J_i^t(\bar{k},\omega) = -\left[\frac{e^2 n_e}{mc}\delta_{ij} + \frac{1}{\hbar c} D_{ij}^t(\bar{k},\omega)\right] A_j^t(\bar{k},\omega) \qquad (A.1)$$

$$(ij = 1,2,3)$$

where $J_i^t(\bar{k},\omega)$ is the Fourier transform of the transverse current $\langle \hat{J}_i^t(\bar{x}t)\rangle$ and $D_{ij}^t(\bar{k}\omega)$ is the F.T. of the retarded transverse current-current response function:

$$D_{ij}^t(\bar{x}-\bar{x}',t-t') = -i\theta(t-t')\langle\phi_o|[\hat{j}_i^{ot}(\bar{x},t),\ \hat{j}_j^{ot}(\vec{x},t)]|\phi_o\rangle$$

$$= \frac{1}{(2\pi)^4}\int d\bar{k}d\omega\ e^{i\bar{k}\cdot(\bar{x}-\bar{x}') - i\omega(t-t')}\ D_{ij}^t(\bar{k},\omega) \qquad (A.2)$$

where $\hat{j}_i^{ot}$ is the transverse part of $\hat{j}_i^o$ defined by (2.44). The vector potential satisfies:

$$A_j^t(\bar{k},\omega) = -\frac{4\pi}{c}\left[\left(\frac{\omega}{c}+i\eta\right)^2 - k^2\right]^{-1} J_j^t(\bar{k},\omega) \qquad (A.3)$$

where the external current has been assumed to vanish. By using (A.3) to eliminate A_j^t in (A.1) the self consistent linear response equation becomes:

$$\left[\left[\left(\frac{\omega}{c}+i\eta\right)^2 - k^2\right]\delta_{ij} - \frac{4\pi e^2 n_e}{mc^2}\delta_{ij} - \frac{4\pi}{\hbar c^2} D_{ij}^t(\bar{k},\omega)\right] J_j^t(\bar{k},\omega)$$

$$= 0 \qquad (A.4)$$

The 3D response function D^t_{ij} is:

$$D^t_{ij}(\bar{k}\omega) = \frac{k^2 e^2 \hbar^3}{(2\pi)^3 m^3} \Pi^{3D}_{ij}(\bar{k}) \int dq \frac{\theta(k_F - q) q^2 \sin^2\theta}{(\omega - \frac{\hbar \bar{k}\cdot\bar{q}}{m} + i\eta)^2 - (\frac{\hbar k^2}{2m})^2}$$

$$= \frac{1}{5} (\frac{\hbar k k_F}{m\omega})^2 \frac{e^2 n_e \hbar}{m} \Pi^{3D}(\bar{k}) + O(k^4) \qquad (A.5)$$

where $k_F^3 = 3\pi^2 n_e$. Now the dispersion for 3D transverse plasmon can be found by using (A.5) in (A.4)

$$\omega^2 = \frac{4\pi e^2 n_e}{m} + c^2 k^2 + \frac{1}{5} (\frac{\hbar k k_F}{m})^2 + O(k^4)$$

which agrees with the previous results of D. Bohm and D. Pines (D. Bohm, 1951) and H. Matsumoto et. al. (H. Matsumoto, 1980).